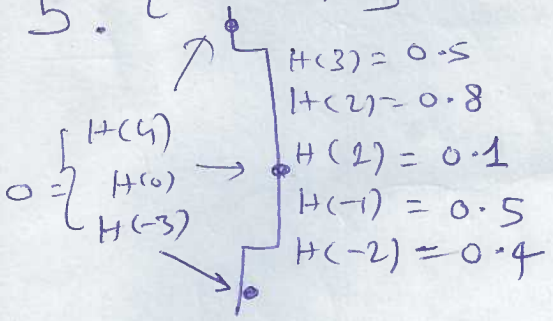
5. [10 marks]



(c) $R = \sum_k \log_2\left(1 + \frac{|H(k)|^2 P(k)}{N_0}\right)$ $\quad N_0 = 0.1$; $P_T = 1.5$ Joules

(a) Uniform Power Allocation $P(k) = \frac{1.5}{5}$; $\forall k$

$\Rightarrow \frac{P(k)}{N_0} = \frac{1.5/5}{0.1} = 3$

$$R = \frac{1}{\log_{10} 2} \times \left[\underbrace{\log_{10}\left(1 + 0.25 \times 3\right)}_{0.48607} \times 2 + \log_{10}(1 + 3 \times 0.64) + \log_{10}(1 + 0.16 \times 3) + \log_{10}(1 + 0.01 \times 3) \right]$$

$\log_{10} 2 \rightarrow 0.3$

$$= \frac{1.13456}{0.3} = 3.769$$

$$R_{UNI} \approx \boxed{3.77 \text{ bits/sec/Hz}} \quad \text{-2-}$$

(b) Zero-forcing Power Allocation

$$P(n) = \frac{K}{|H[n]|^2} \Rightarrow K\left[\underbrace{\frac{1}{0.25}}_{8} \times 2 + \frac{1}{0.64} + \underbrace{\frac{1}{0.01}}_{100} + \frac{1}{0.16}\right] = \frac{3}{2} \quad P_T$$

$108 + \frac{125}{16}$

$\therefore K \times 115.87 = 1.5$

$\Rightarrow K = 0.012952$;

$$\Rightarrow R_{ZF} = \frac{1}{\log_{10} 2}\left[\log_{10}\left(1 + \underbrace{\frac{K}{N_0}}_{1.12952}\right) \times 5\right] = 0.8785$$

$\boxed{R_{ZF} \approx 0.88 \text{ bits/sec/Hz}}$ -2-

(c) Optimal Water Pouring Allocation N_0

$$\sum_k \left(\frac{1}{\lambda} - \frac{0.1}{|H(k)|^2}\right) = \frac{3}{2} \quad P_T$$

$$\Rightarrow \frac{5}{\lambda} = \frac{3}{2} + \left[\underbrace{\frac{0.1}{0.25}}_{0.8} \times 2 + \frac{0.1}{0.64} + \underbrace{\frac{0.1}{0.01}}_{10} + \frac{0.1}{0.16}\right]$$

$$\therefore \lambda = \frac{5}{13.08125} = \boxed{0.382} \quad \text{-1-} \qquad \left(\& \ \frac{1}{\lambda} = 2.616\right)$$

5 (c) part cont'd

Company

$\frac{1}{\lambda} \Rightarrow 2.616$ mW

$\frac{0.1}{0.25} = \frac{2}{5} = 0.4$ ✓

$\frac{0.1}{0.64} = \frac{10}{64}$ ✓

$\frac{0.1}{0.16} = \frac{10}{16}$ ✓

$\frac{0.1}{0.01} = 10$ which is $> \frac{1}{\lambda}$ ✓ don't use ✗

$\therefore P(1) = 0$; → -1-

Now Re-compute λ for Non-Zero terms

$$\Rightarrow \sum_{k} \left(\frac{1}{\lambda} - \frac{0.1}{|H(k)|^2} \right) = 3/2$$

$$\Rightarrow \frac{4}{\lambda} = 0.1 \left[\underbrace{\frac{1}{0.25} \times 2}_{8} + \frac{1}{0.64} + \frac{1}{0.16} \right] + 1.5$$

$\Rightarrow \frac{4}{\lambda} = 3.08125$ ~~2.3975~~ $\Rightarrow \frac{1}{\lambda} = 0.7703$ & $\lambda_{new} = 1.2982$ -1-

$\therefore P(3) = 0.3703$

$P(2) = 0.6140$

$P(1) = 0$

Same → $P(-1) = 0.3703$

$P(-2) = 0.1453$

Power allocation → -0.5-

(d)
$C > R_{uni} > R_{zf}$
All 3 equal iff
$|H(k)|^2 = \alpha + k$ (single tap channel) → -1.5-

$$C = \frac{1}{\log_{10} 2} \left[\log_{10}\left(1 + \frac{0.25 \times 0.3703}{0.1}\right) \times 2 + \log_{10}\left(1 + \frac{0.64 \times 0.6140}{0.1}\right) + \log_{10}\left(1 + \frac{0.16 \times 0.1453}{0.1}\right) \right] = 4.4939 \text{ b/s/Hz}$$

-1-