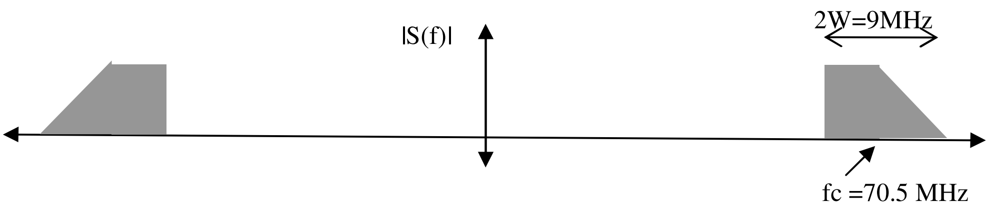
EE-5060 Communication Techniques

Aug. 2010 **Tutorial #1** KG / IITM

1. A low-pass signal of one-sided bandwidth of W=1.25MHz is sent as a DSB-SC signal. If the receiver uses an IF sampling scheme, with center frequency f_{IF} = 7MHz, determine the least sampling rate required by using the following constructions:
(a) "Growing the assumed band-width" on both sides
(b) "Growing the assumed band-width" only towards the origin
(c) "Growing the assumed band-width" only away from the origin
Draw a rough sketch of the spectrum of the sampled sequence between -8MHz and +8MHz for each of the above cases.

2. In the above problem, if we are to use single-side band signaling, where only the upper-side band is to be sent, what will be your answer to

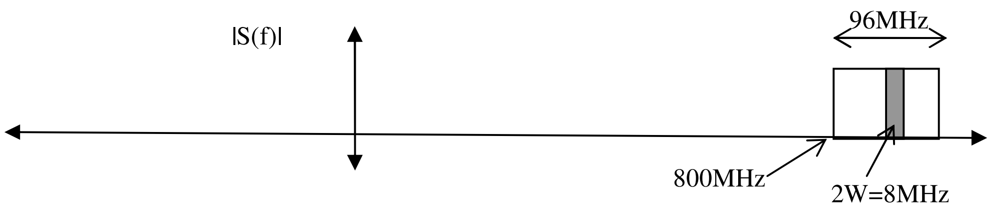
3. For the QCM signal with magnitude response as below, find the least possible band-pass sampling rate. Make a rough plot of the frequency response of the sampled sequence around 0Hz.



4. Repeat Pbm.3 by considering the construction where the assumed spectrum is "grown only away from the origin".

5. In Pbm.3, assume that the received signal has a phase offset of θ radians; in other words, $s(t) = m_1(t)Cos(2\pi f_c t + \theta) + m_2(t)Sin(2\pi f_c t + \theta)$. Now, what will be the time-domain representation of the sampled sequence? For the special case when θ = $\pi/2$, what will be the samples of the received signal?

6. A dozen DSB-SC signals of one-sided (low-pass) bandwidth W = 4MHz are present between 800MHz and 896MHz, as shown below. Describe the operations (sampling, rate-conversion, filtering) that you need to do to recover Nyquist rate samples of the 7^{th} DSB-SC signal (i.e., the signal present between 848Mz and 856MHz).



7. X is a (random) variable with an uniform probability density function (pdf) between the two limits (a, b) specified simply as follows: $f_X(x) = 1/(b-a), if a \le x \le b; else f_X(x) = 0.$ The mean value of X is defined by $m_X = E[X] = \int_a^b x f_X(x)dx$ and the power or variance defined by $\sigma_X = E[(X - m_X)^2]$. Using this, show that the variance of the uniformly distributed X is given by $\sigma_X = (b-a)^2/12$.

8. A WSS random process has an acf given by $R_X(\tau) = \frac{A^2}{2} e^{-|\tau|} Cos(2\pi f_0 \tau)$. Assume that the random process never exceeds 6 in magnitude, and that A=6.
(a) How many uniform quantization levels are required to provide an SQNR of at least 40 dB?
(b) If we want to increase the minimum SQNR to 60 dB, how should we required the number of quantization levels change?
(c) If $f_0 = 1Mhz$, what is the bit rate you will require to send the quantized samples in both of above cases.

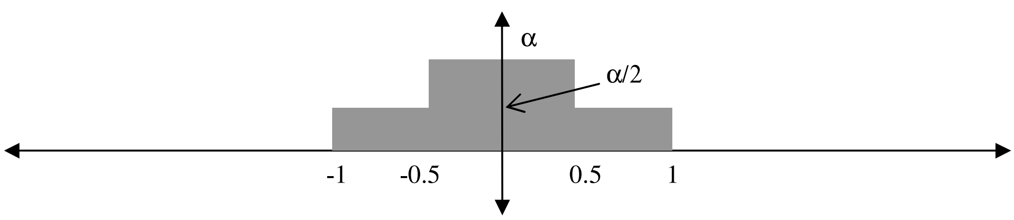
9. Llyod-Max Non-uniform Quantization: Assume that a random signal x(t) with x=x(t=t$_0$), follows a probability density function $f_X(x)$. and the objective is to find the N-1 signal levels $\{a_1, a_2, \cdots, a_{N-1}\}$and the corresponding N quantized values $\{\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_N\}$ such that the quantization noise variance given by

$$E_q = E[(x - x_q)^2] = \int_{-\infty}^{a_1} (x - \hat{x}_1)^2 f_X(x)\, dx + \sum_{i=2}^{N-1} \int_{a_{i-1}}^{a_i} (x - \hat{x}_i)^2 f_X(x)\, dx + \int_{a_{N-1}}^{\infty} (x - \hat{x}_N)^2 f_X(x)\, dx$$

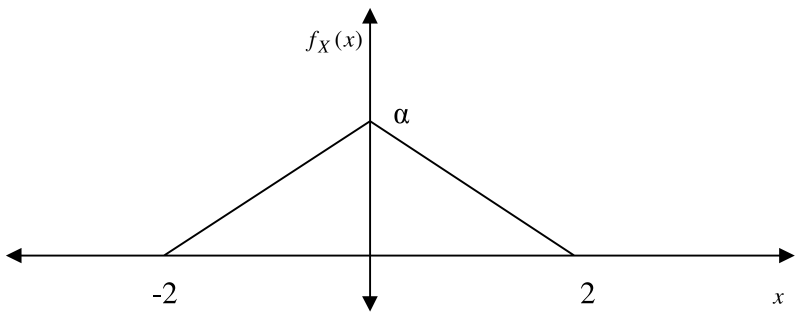
is minimized. In other words, differentiate E_q w.r.t $\{a_1, \cdots, a_{N-1}\}$ and $\{\hat{x}_1, \cdots, \hat{x}_N\}$, equate them to zero, and show the following:

$$(i)\, a_i = \frac{\hat{x}_i + \hat{x}_{i+1}}{2} \quad \text{and} \quad (ii)\, \hat{x}_i = \frac{\int_{a_{i-1}}^{a_i} x\, f_X(x)\, dx}{\int_{a_{i-1}}^{a_i} f_X(x)\, dx}$$

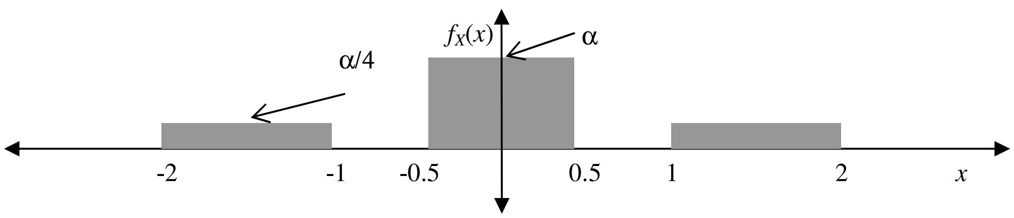
10. The Llyod-Max quantizer indicates that the signal level should be the mid-point of the quantization interval (see (i)), and that the quantized value is the "centroid" of the corresponding interval (see (ii)). However, a practical method to construct this non-uniform quantizer is given by the iterative method below for a k-bit quantizer where N=2^k:
Step 1: choose N-1 uniform intervals $\{a_1, a_2, \cdots, a_{N-1}\}$
Step 2: find the corresponding centroids $\{\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_N\}$using ($ii$)
Step 3: re-compute $\{a_i\}$ using (ii) in (i)
Iterate between steps 2 and 3 until "convergence"
Use the above procedure for 3 iterations to define the 2-bit LM quantizer for the below source pdf. Compare the E_q obtained between the uniform quantizer (mid-tread) and the LM quantizer.



11. A source X with a triangular pdf as below is to be quantized to 4 levels:
(a) Find α.
(b) Assume a simple uniform quantizer (the obvious choice being $a_1=-1$, $\Delta=1$), find the pdf of the quantization error given by $E=X-X_q$ where X_q is quantized value.
(c) What is the SQNR for the uniform 4 level quantizer?
(d) Perform 3 iterations for the Llyod-Max non-uniform quantizer, starting with the initial decision regions as defined in part (b). What is the new SQNR?

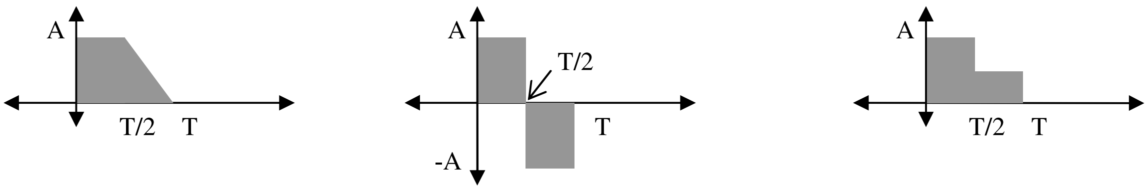


12. A signal x, described by the pdf $f_X(x)$ as below, is to be quantized:
(a) For what value of α is this a valid pdf ?
(b) Find the quantization error variance E_q for a 2-bit uniform quantizer with left-most quantization interval $a_1=-1$, and $\Delta=1$. (*Hint*: use these to define the other quantization intervals, and the quantization levels $\{\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_N\}$.
(c) Now, define the non-uniform 2-bit quantizer for this pdf, where the above uniform quantizer can be taken as the initial guess. Use the practical approach to implement the Llyod-Max rule, and run it over 3 iterations. How does the E_q of this compare to case (b)?



13. From "Digital Telephony" J.C.Bellamy, 3rd Ed., pp. 158-160: Problems **3.1, 3.2, 3.3, 3.4, 3.5, 3.8*, 3.14*, 3.16*,3.17*, 3.18,** and **3.19.**

14. Draw the output of the matched filter over 3 symbol intervals for the bit-sequence 1,-1,1 for each of the below choices of the pulse-shape $g(t)$:



15. For a bit-stream with 8 consecutive bits given by 1,-1,1,-1,-1,-1,1,1, make a rough plot of the following line-codes: (a) NRZ (b) AMI (c) Split-phase Manchester (d) CMI (e) Differential Modulation (f) Differential Split-phase Manchester (g) B3ZS (or HDB3)

16. A state-dependent Miller code of rate 1/2 is defined by the following rule:
(a) input "0" → output di-bit is: *x* 0 where *x*=0 if preceding input bit is "1" (and "*x*=1" otherwise)
(b) input "1" → output di-bit is: 0 1
Use this code on the bit-sequence in Pbm.10. What properties of the Miller code do you observe?