EE 504 – Adaptive Signal Processing

(Adaptive Equalization for ISI channels)

Due Date: Oct. 15, 2007 **Take-Home Assignment-I** **10 Marks**

Instructions: Your soft-copy submission must be only a *single* PDF or MS-Word file, incorporating all the plots, tables , and text matter. The file should be named "ee504_nameofthestudent.pdf/doc " and be emailed to the TA's R. Lakshminarayanan (rln@tenet.res.in) and T. Venkatachalam (venkinaren@tenet.res.in), and also cc'ed to giri@tenet.res.in. Independent effort is expected from each student. You may be called to explain your work if deemed necessary. Only submissions received on or before Monday 6.00pm, October 15, 2007, will be graded.

A real, i.i.d. sequence $\{I(k)\}$ where each element is drawn from a bipolar constellation with $\sigma_I^2 = E[I^2(k)] = 1$, is filtered by a FIR filter $F(z) = \frac{1}{c}(1 - \sqrt{2}rz^{-1} + r^2z^{-2})$ as shown in the figure below. The scaling constant ($1/c$) is chosen so as to make $F(z)$ have unit gain, and consider $r=0.999$ in the definition for $F(z)$. The resultant output $x(k)$ is corrupted by a real Gaussian noise sequence $\{v(k)\}$ with variance σ_v^2 to finally yield the measurements $u(k)=x(k)+v(k)$. Here, $\{I(k)\}$ and $\{v(k)\}$ are mutually uncorrelated, and take $u(k)$ and $I(k) = 0$ for $k \le 0$. The received SNR (in dB scale) is then equal to $10\log_{10}(1/\sigma_v^2)$, where the SNR can be conveniently varied by changing the noise variance. The desired response is $d(k)=I(k-\Delta)$, where $0 \le \Delta \le L$ is the decoding delay.

1. MMSE Equaliser: Design a Wiener equalizer $\mathbf{w}_{MMSE}$ of order L and decoding delay Δ. Tabulate MMSE, $J_{min}(L,\Delta,\text{SNR})$ for (L,Δ,SNR) given by: **a.** (3,0, 30dB) **b.** (3,0,15dB) **c.** (10,0,15dB) **d.** (10,3,15dB) **e.** (50,3,15dB) **f.** (50,3,30dB)

2. SDA Equaliser: Design an adaptive equalizer using the Steepest Descent Rule (using the values of **R** and **p** determined in the above part). Starting with $\mathbf{w}(0)=[0,0,\ldots 0]^T$, plot the learning curve in the dB scale for cases **a.** thro **f.** as above (by ensemble averaging $10\log_{10}(e^2(k))$ over 25 independent trials, each trial over $k=1,2,\ldots,N$, with $N=5000$ symbols) of the SDA equalizer. Use $\mu = 0.01$.

3. LMS Equaliser: For the Least Mean Squares algorithm, repeat the tasks specified for the SDA above. Use $\mu = 0.01$.

4. n-LMS Equaliser: For the normalized-LMS algorithm, repeat the tasks specified for the SDA above. Use $\mu = 2.0$.

5. Use $r=0.800$: For this new $F(z)$, determine $J_{min}(L,\Delta,\text{SNR})$ for $L=6$, $\Delta=1$, and SNR=20dB. For this case, plot for 25 independent trials of $N=1000$ symbols each, the ensemble averaged learning curves (on the same plot) for SDA, LMS, and n-LMS. Choose the "best-possible" gain constant in each case, and justify your choice.

Hint: The ensemble-averaged learning-curves can be made to look "neater" by also smoothing the curves with a moving average filter. For example, an 11[th] order filter with each coefficient equal to (1/11) will be a good choice.