Quiz 1 Solutions

September 30, 2007

1.

$$
\begin{align}
E[X|Y] &= E[X] + R_{xy}R_y^{-1}(Y - E[Y]) \tag{1}\\
W &= \begin{bmatrix} Y \\ Z \end{bmatrix} \tag{2}
\end{align}
$$

$$
E[W] = \begin{bmatrix} E[Y] \\ E[Z] \end{bmatrix} \tag{3}
$$

$$
\begin{align}
E[X|W] &= E[X] + R_{xw}R_w^{-1}(W - E[W]) \tag{4}\\
R_{xw} &= [R_{xy}|R_{xz}] \tag{5}\\
R_w &= \left[\begin{array}{c|c} R_y & 0 \\ \hline 0 & R_z \end{array}\right] \tag{6}
\end{align}
$$

$$
\begin{align}
E[X|W] &= E[X] + R_{xy}R_y^{-1}(Y - E[Y]) + R_{yz}R_z^{-1}(Z - E[Z]) \tag{7}\\
&= E[X|Y] + E[X|Z] - E[X] \tag{8}\\
E[X|Y,Z] &= E[X|Y] + E[X|Z] - E[X] \tag{9}
\end{align}
$$

2.

$$
\begin{align}
q(k) &= v(k) * g(k) \tag{10}\\
q(k) &= 0.9q(k-1) - 0.81q(k-2) + v(k) \tag{11}\\
u(k) &= q(k) + w(k) \tag{12}\\
d(k) &= u(k) \tag{13}\\
U(k-1) &= \begin{bmatrix} u(k-1) \\ u(k-2) \end{bmatrix} \tag{14}
\end{align}
$$

$$
\begin{align}
w_{opt} &= R^{-1}p \tag{15}\\
R &= E[U(k-1)U^H(k-1)] \tag{16}\\
p &= E[d^*U(k-1)] \tag{17}
\end{align}
$$

$$
\begin{align}
r_q(0) &= E[q^2(k)] \tag{18}\\
&= 0.9^2 E[q^2(k-1)] + 0.81^2 E[q^2(k-2)] + \sigma_v^2 - 2(0.9)(0.81)E[q(k-1)q(k-2)] \tag{19}\\
&= 0.9^2 r_q(0) + 0.81^2 r_q(0) + \sigma_v^2 - 2(0.9)(0.81)r_q(1) \tag{20}\\
\sigma_v^2 &= r_q(0)(1 - 0.9^2 - 0.81^2) + 2(0.9)(0.81)r_q(1) \tag{21}
\end{align}
$$

$$
\begin{align}
r_q(1) &= E[q(k)q(k-1)] \tag{22}\\
&= 0.9r_q(0) - 0.81r_q(1) \tag{23}\\
r_q(1)(1 + 0.81) &= 0.9r_q(0) \tag{24}\\
r_q(1) &= (0.9/1.81)r_q(0) \tag{25}\\
&\tag{26}
\end{align}
$$

$$
\begin{align}
r_q(2) &= E[q(k)q(k-2)] \tag{27}\\
&= 0.9r_q(1) - 0.81r_q(0) \tag{28}\\
&= (0.9^2)/1.81)r_q(0) - 0.81r_q(0) \tag{29}
\end{align}
$$

$$
\begin{align}
r_q(0) &= 3.863 \tag{30}\\
r_q(1) &= 1.921 \tag{31}\\
r_q(2) &= -1.4003 \tag{32}\\
&\tag{33}
\end{align}
$$

$$
R = \begin{bmatrix} 3.863 + 0.4 & 1.921 \\ 1.921 & 3.863 + 0.4 \end{bmatrix} \tag{34}
$$

$$
R = \begin{bmatrix} 4.263 & 1.921 \\ 1.921 & 4.263 \end{bmatrix} \tag{35}
$$

$$
p = \begin{bmatrix} 1.921 \\ -1.4003 \end{bmatrix} \tag{36}
$$

$$
\begin{align}
w_{opt} &= R^{-1}p \tag{37}\\
&= \begin{bmatrix} 0.751 \\ -0.667 \end{bmatrix} \tag{38}
\end{align}
$$

$$
\begin{align}
\sigma_w^2 &= 0 \tag{39}\\
R &= \begin{bmatrix} 3.863 & 1.921 \\ 1.921 & 3.863 \end{bmatrix} \tag{40}
\end{align}
$$

$$
\begin{align}
w_{opt} &= R^{-1}p \tag{41}\\
&= \begin{bmatrix} 0.90 \\ -0.81 \end{bmatrix} \tag{42}
\end{align}
$$

$$
\begin{align}
q(k) &= 0.9q(k-1) - 0.81q(k-2) + v(k) \tag{43}\\
&\tag{44}
\end{align}
$$

$$
\begin{align}
J_{min} &= \sigma_d^2 - w_{opt}^T p \tag{45}\\
&= \sigma_u^2 - w_{opt}^T p \tag{46}\\
w_{opt} &= \begin{bmatrix} 0.751 \\ -0.667 \end{bmatrix} \tag{47}
\end{align}
$$

$$
\begin{align}
J_{min} &= r_q(0) + \sigma_w^2 - w_{opt}^T p \tag{48}\\
&= 3.863 + 0.4 - w_{opt}^T p \tag{49}\\
&= 1.886 \tag{50}\\
\sigma_w^2 &= 0 \tag{51}\\
J_{min} &= 3.863 - w_{opt}^T p \tag{52}\\
w_{opt} &= \begin{bmatrix} 0.90 \\ -0.81 \end{bmatrix} \tag{53}
\end{align}
$$

$$J_{min} \simeq 1.0 \tag{54}$$
$$= \sigma_v^2 \tag{55}$$

3.

$$R = \begin{bmatrix} 2 & (1+j)/\sqrt{2} \\ (1-j)/\sqrt{2} & 2 \end{bmatrix} \tag{56}$$

$$p = \begin{bmatrix} 1+j \\ 0 \end{bmatrix} \tag{57}$$

$$R = Q\Lambda Q^H \tag{58}$$
$$Q = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ (-1+j)/2 & (-1+j)/2 \end{bmatrix} \tag{59}$$

$$\Lambda = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \tag{60}$$

$$R^{-1} = Q\Lambda^{-1}Q^H \tag{61}$$
$$\Lambda^{-1} = \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix} \tag{62}$$

$$v(n+1) = (I - \mu\Lambda)^n v(0) \tag{63}$$
$$v(n) = Q^H(w_{opt} - w(n)) \tag{64}$$
$$v(0) = Q^H w_{opt} \tag{65}$$

$$v(0) = \begin{bmatrix} -0.2357 - j0.2357 \\ (1+j)/\sqrt{2} \end{bmatrix} \tag{66}$$

$$v(9) = (I - \mu\Lambda)^8 v(0) \tag{67}$$
$$= \begin{bmatrix} -0.2357 - j0.2357 \\ 0.0001 + j0.0001 \end{bmatrix} \tag{68}$$

$$v(9) = Q^H(w_{opt} - w(9)) \tag{69}$$
$$Qv(9) = w_{opt} - w(9) \tag{70}$$
$$w(9) = w_{opt} - Qv(9) \tag{71}$$
$$= \begin{bmatrix} 0.5 + j0.5 \\ -1/\sqrt{2} \end{bmatrix} \tag{72}$$

$$M = 2 \tag{73}$$
$$Misadj = (\mu/2)tr(R) \tag{74}$$
$$= (\mu/2)Mr(0) \tag{75}$$
$$0.2 = (\mu/2)(2)(2) \tag{76}$$
$$\mu = 0.1 \tag{77}$$