Department of Electrical Engineering, IIT Madras

EE 504 : Adaptive Signal Processing

Quiz-1

20 Marks **Sept. 2007**

1. [5 marks] Let **x**, **y**, and **z** be jointly Gaussian random vectors, of dimensions Lx1, Mx1, and Nx1, respectively. If **y** and **z** are uncorrelated, then prove that

$$E[\mathbf{x} \mid \mathbf{y},\mathbf{z}] = E[\mathbf{x} \mid \mathbf{y}] + E[\mathbf{x} \mid \mathbf{z}] - E[\mathbf{x}] \quad (1.1)$$

Hint: Define an appended vector $\mathbf{w} = \begin{bmatrix} \mathbf{y} \\ \mathbf{z} \end{bmatrix}$ and use it in the expression for the conditional mean.

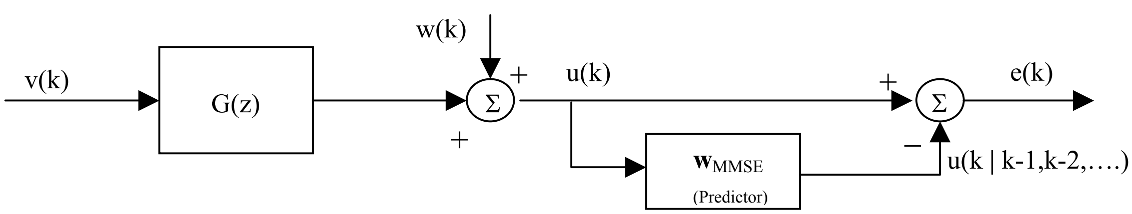
2. [7 marks] Consider the linear MMSE estimation problem below where the measurements {u(k)} are obtained by passing a white sequence {v(k)} with $\sigma_v^2 = 1.0$ through an infinite impulse response (IIR) transfer function *G(z)* given by

$$G(z) = \frac{1}{(1 - 0.90z^{-1} + 0.81z^{-2})} \quad (1.2)$$

and the resultant output is further corrupted by a noise component w(k) with $\sigma_w^2 = 0.40$, as shown below.

(a) If the one-step linear MMSE predictor (which uses u(k-1), u(k-2),….etc., to predict u(k)) is to have an order M=2, find it's coefficients $\mathbf{w}_{MMSE} = [w_1\ w_2]^T$. It can be assumed that {v(k)} and {w(k)} are mutually uncorrelated. *Hint:* First determine **R** and **p** and set up the Wiener-Hopf equations.

(b) What is the J_{min} for this predictor?



3. [5+3=8 marks] Given that $E[\mathbf{u}(n)\mathbf{u}^H(n)] = \mathbf{R} = \begin{bmatrix} 2 & \frac{1+j}{\sqrt{2}} \\ \frac{1-j}{\sqrt{2}} & 2 \end{bmatrix}$ and $E[\mathbf{u}(n)d^*(n)] = \mathbf{p} = \begin{bmatrix} 1+j \\ 0 \end{bmatrix}$ answer the following questions

:

a. Consider using the Steepest Descent Algorithm (SDA) starting with $\mathbf{w}(0)=[0\ \ 0]^H$. What will be the value of $\mathbf{w}(9)$ if the largest possible μ was used by the SDA?.

b. Instead, if the (conventional) Least Mean Squares (LMS) algorithm is used with order M=2, what is the value of gain constant μ that will result in only a 20% mis-adjustment at steady state?