Department of Electrical Engineering

EE-5040 : Adaptive Signal Processing

Nov. 2011 **Tutorial #3** KG/IITM

1. In a certain non-stationary model, the measurements are given by u(n)=x(n)+v(n) where the signal component x(n) is the obtained when a white, driving noise sequence p(n) with variance 1.0 is filtered by an pole-zero response $F(z)=(1+0.5z^{-1}) / (1-0.6z^{-1})$.The additive noise component v(n) is also white with variance $\sigma_v^2 = 0.30$, and is mutually uncorrelated with p(n).
These measurement are filtered by an M=7 order FIR estimator **w** to produce y(n) such that $E[(d(n)-y(n))^2]$ is minimized. The desired response follows a non-stationary model where $d(n)=\mathbf{u}^T(n)\mathbf{w}_o(n)+z(n)$, where the time-varying optimum weight follows a random walk model $\mathbf{w}_o(n)=\mathbf{w}_o(n-1)+\mathbf{q}(n)$, where $E[\mathbf{q}\mathbf{q}^T] = \mathbf{Q} = 0.20\ \mathbf{I}_{M\times M}$. An LMS algorithm is used to estimate and track this time-varying model.

(a) If a step size $\mu=0.01$ is used, what will be the tracking excess MSE of the LMS algorithm for M=7? (Use the small step-size approximation if appropriate).

(b) For what choice of $\mu=\mu_{opt}$ will the LMS algorithm have the "best" tradeoff between convergence and tracking performances?

2. For the simple 1st order scalar model given by $x(n)=\phi x(n-1)+\alpha v_1(n-1)$ and $y(n)=cx(n)+v_2(n)$, with the state noise $v_1(n)$ with variance q_1 and the measurement noise $v_2(n)$ with variance q_2, show that:
(a) Asymptotic covariance of the signal x(n) obtained by solving the Lyupunov equation is given by $\bar{p}_x = \alpha^2 q_1 / (1-\phi^2)$.

(b) Thus, the asymptotic signal to noise ratio (SNR) at steady state is given by $SNR = \frac{c^2\alpha^2}{(1-\phi^2)}\left(\frac{q_1}{q_2}\right)$

where the last term $\left(\frac{q_1}{q_2}\right)$ is the SNR tuning parameter.

3. Consider the scalar Kalman Filter model discussed in class, namely by $x(n)=x(n-1)+v_1(n-1)$ and $y(n)=x(n)+v_2(n)$, with $\sigma_1^2=q_1=20$ and $\sigma_2^2=q_2=5$. Get the expression for the steady state value of the error covariance, namely $\lim_{n\to\infty} k(n,n) = \bar{k}$, and using this find the value of the steady state Kalman gain $\lim_{n\to\infty} g(n) = \bar{g}$. Using this $\bar{g}$ show that the filtered update equation corresponds to a stable system, with the pole well inside the unit circle (U.C.), even though the state equation had a pole on the U.C.! Comment on your answer.

4. Consider the issue of estimating two state variables $x_1(n)$ and $x_2(n)$, where $x_1(n) = \alpha_1 x_1(n-1) + \alpha_2 x_1(n-2) + v_1(n)$, and $x_2(n) = \beta\, x_2(n-1) + v_2(n)$. Measurement equation is given by $y(n) = c_1(n)\, x_1(n)$

+ $c_2(n)\, x_1(n-1) + c_3(n)\, x_2(n) + w(n)$, where $v_1(n)$, $v_2(n)$, and $w(n)$ are mutually uncorrelated, Gaussian, and with variances σ_1^2, σ_2^2, and σ_w^2, respectively.

(a) Set up a state-space model, clearly specifying the state-transition matrix **F**, and the measurement matrix (or vector) **C**.

(b) For the above model, given the initial guess for the filtered error covariance matrix is $K(0,0)=\gamma I$, use the predictor-corrector form for the Kalman filter to specify the expression for the Kalman gain $\mathbf{g}(n)$ at time n=1 (in terms of the other given quantities).

5. A scalar state-space model is give by $x(n)=0.9x(n-1)+v_1(n-1)$ and $y(n)=x(n)+v_2(n)$, where $\sigma_1^2 = 1$, and $\sigma_2^2 = 0.4$.

(a) Find the steady state value of the error covariance, namely, $\lim_{n\to\infty} k(n,n) = \bar{k}$.

(b) Starting with $k(0,0)=40$, find (i) $k(2,1)$ and (ii) $k(2,2)$.