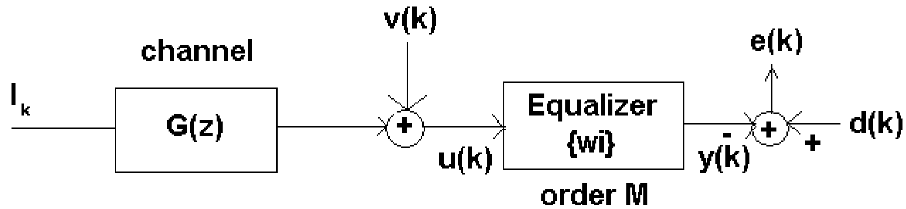
Department of Electrical Engineering

EE-5040 : Adaptive Signal Processing

Sept. 2011 **Tutorial #2** KG/IITM

1. In the figure below, the input $\{I_k\}$ is i.i.d with $E[I_k^2]=1$ and $P[I_k=+1]=1/2=P[I_k=-1]$. The AWGN v(k) has variance $\sigma_v^2=0.2$ and $E[I_k\, v(i)]=0$ for all k,i.



The channel $G(z)=1-z^{-1}+0.5\,z^{-2}$ and the equalizer has an order equal to M. For the following situations, compute manually **R**, **p**, and eventually $\mathbf{w}_{opt}=\mathbf{R}^{-1}\mathbf{p}$.

a) M=2, d(k)=I(k)
b) M=2, d(k)=I(k-1)
c) M=2, d(k)=I(k-2)
d) What is the J_{min} in each of the above cases?
e) Now, consider a Decision Feedback Equaliser (DFE) with M=2 taps in the feedforward section and 2 taps in the feedback section. For each of the cases in (a), (b), and (c), what will be the resultant **R** and **p** in this case?

2. A real WSS process u[n] is to be filtered by a 2-tap estimator **w** such that $E\{|y[n]|^2\}$ is minimized where $y[n]=\mathbf{w}^T\mathbf{u}[n]$, subject to $\mathbf{w}^T\mathbf{g}=1$ where $\mathbf{g}=[1\ -1]^T$.

a) Given that r[0]=5, and r[1]=2, use the Lagrange multiplier technique to determine **w**. What is the value of the Lagrangian (constant)?
b) Repeat part (a) with the alternative constraint $\mathbf{w}^T\mathbf{w}=1$. Specify all values of **w**.

3. Steepest Descent Algorithm (SDA) is used to find $\mathbf{w_o}$ with $r[0]=2, r[1]=(1+j)/\sqrt{2}=r^*[-1]$ and $\mathbf{p}=[1+j\ 0]^T$.

a) Determine the μ that can be used to provide fastest convergence while ensuring stability.
b) Starting with $\mathbf{w}=[0\ 0]^T$ and with μ from above, find $\mathbf{v}^H[25]\mathbf{v}[25]$ where $\mathbf{v}[n]=\mathbf{Q}^H(\mathbf{w}[n]-\underline{\mathbf{w}_o})$.

4. Consider the following 7 measurements {u[k]} and the desired signal {d[k]}

k	1	2	3	4	5	6	7
u[k]	3	3	-3	-3	3	-3	3
d[k]	-2.9	5.1	-1.4	-4.9	1.3	-1	0.9

a) Find the least squares estimate $\mathbf{w}_{LS}=[w_0\ w_1]^T$.
b) Starting with $\mathbf{P}(0)=10\mathbf{I}$ and $\mathbf{w}_{RLS}(0)=[0\ 0]^T$ find $\mathbf{w}_{RLS}(3)$, $\mathbf{P}(3)$, and e(3)?

5. Solve problems from "Adaptive filter theory " by S.Haykin 3rd ed.

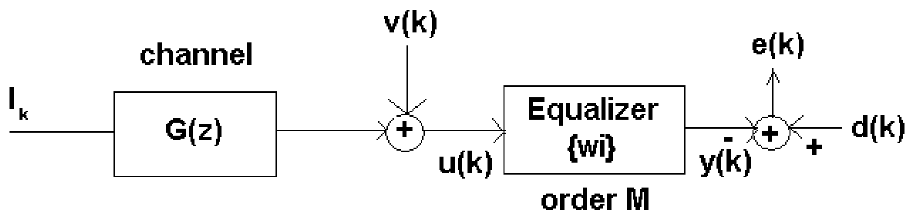
i. pp. 533-535 (Ch 11): 1,2,3,8,9,10.
ii. pp. 587-588 (Ch 13): 1,3,4,5,6.

6. A uniform real i.i.d sequence {d[k]} with $E\{|d[k]|^2\}=1$ is filtered by $H(z)= 1-0.5z^{-1}+(1/3)\ z^{-3}$ and the resulting output is corrupted by a coloured noise which is a result of AWGN filtered by $1+0.8z^{-1}$ to give the measurements {u[k]} where the variance of the AWGN is 0.25 and this AWGN is uncorrelated with {d[k]}.

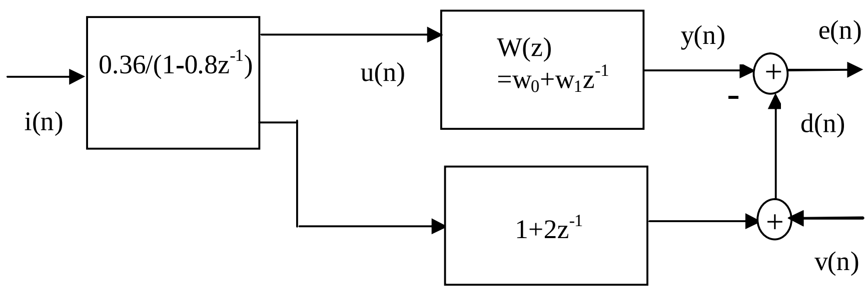
a) Find $\mathbf{R}_{uu}$ of size 3×3.

b) Find a 3×1 $\mathbf{p}=E\{\mathbf{u}[k]d[k-\Delta]\}$ for (i)$\Delta=1$,(ii)$\Delta=4$.

7. In the figure below, the input $\{I_k\}$ is i.i.d with $E[I_k^2]=1$ and $P[I_k=+1]=1/2= P[I_k=-1]$. The AWGN v(k) has variance $\sigma_v^2=1$ and $E[I_k\ v(i)]=0$ for all k,I, and the channel response $G(z)=1/(1-0.8z^{-1})$. If $\mathbf{w}$ is of order M=2, find $\mathbf{w_o}$ for (a) $\sigma_v^2=1$, (b) $\sigma_v^2=0.3$ and (c) $\sigma_v^2=0$.



8. Consider the measurement model below, where a linear MSE (Wiener) estimator is to be defined for the equalizer W(z) which will minimize $E[e^2(n)]$. Here, input i(n) is white noise with unit variance, and additive noise v(n) has variance $\sigma_v^2 = 0.10$ and is uncorrelated with i(n).

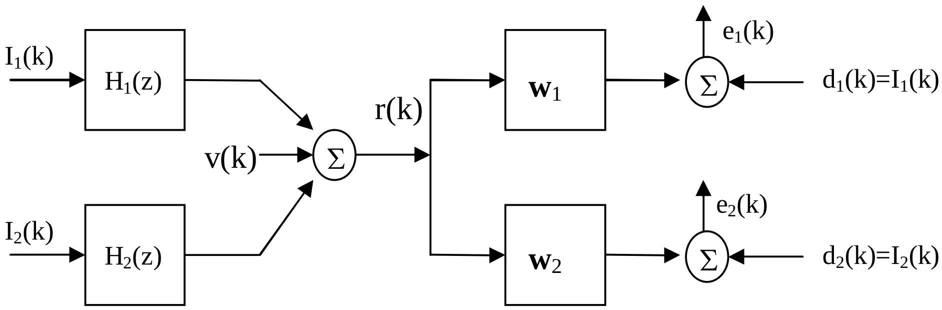


(a) Find the correlation matrix $\mathbf{R}$ and the cross-correlation vector $\mathbf{p}$.

(b) What is the LMSE estimate for W(z)?

(c) What is the Jmin for this LMSE problem?

9. Two mutually uncorrelated i.i.d bipolar bit-streams $I_1(k)$ and $I_2(k)$ with variances σ_1^2 and σ_2^2 are filtered by $H_1(z)=0.8+z^{-1}$ and $H_2(z)=1-0.7z^{-1}$, respectively. Their filtered outputs get added to an AWGN process v(k) with variance σ_v^2, so as the produce the measurements r(k) as shown in the figure below. The receiver uses two equalizers, $\mathbf{w}_1$ and $\mathbf{w}_2$, to recover (estimate) $I_1(k)$ and $I_2(k)$, respectively.



(a) If $\sigma_1^2 = 3$, and $\sigma_2^2 = \sigma_v^2 = 0$, specify the MMSE estimate of $\mathbf{w}_1$ of order M=5. *Hint*: At infinite (sufficiently high) SNR, MMSE and zero-forcing estimators are identical.

(b) Now if $\sigma_1^2 = 1, \sigma_2^2 = 0.8$, and $\sigma_v^2 = 0.2$, which of the above two equalizers will give a better estimate of the corresponding data symbol? *Hint*: That equalizer providing the lower J_{min} will be the better one.

10. Given r(0)=3 and r(1)=r(-1)=1 and $\mathbf{p}=[1\ 0]^T$ and SDA is used with μ=0.25. Starting with $\mathbf{w}_0=\mathbf{0}$ and order M=2 perform SDA iterations (*Hint: cleverly !!*) to find $\mathbf{w}[25]$. Also find the μ which gives fastest convergence.

11. Show that the Newton method can also be written as $\mathbf{w}(n) = \mathbf{w}_O + (1-2\mu)^n(\mathbf{w}(0) - \mathbf{w}_O)$. Also, choosing μ=1/2 is this expression ensures 1-step convergence. Where is the catch?

12. Show that another equivalent form for the LMS update equation is given by
$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu\, e(n)\mathbf{u}^*(n)$ where error $e(n) = d(n) - w^T(n)u(n)$, and "T" is the transpose operation.

13. After defining the weight-error vector by $\boldsymbol{\varepsilon}(n) = \mathbf{w}_O - \mathbf{w}(n)$, show that the LMS weight update equation can be re-written as $\boldsymbol{\varepsilon}(n) = [\mathbf{I} - \mu\mathbf{u}(n)\mathbf{u}^H(n)]\boldsymbol{\varepsilon}(n) - \mu\mathbf{u}(n)e_O^*(n)$ where the estimation error produced by the optimum Wiener filter is given by $e_O(n) = d(n) - \mathbf{w}_O^H\mathbf{u}(n)$.

14. Consider the figure used in Pbm.**7** where we are interested in formulating the channel and the corresponding equalizer using LS estimators with the following measurements:

k	1	2	3	4	5	6	7
u[k]	1.2	-0.3	0.2	1.5	-0.4	0.1	0.9
I[k]	1	-1	1	1	-1	1	--

Assume Δ=0 and I(k)=0 for n<1 and n>6. The channel has only 2 taps.

(a) Find its LS estimate using the covariance formulation. Also find LS estimate of equalizer. Observing the convolution of the channel and equalizer, comment on a good choice of the decoding delay Δ.

(b) Repeat the above using instead the auto-correlation formulation for the LS estimate.

15. From "Fundamentals of Adaptive Filtering" by A.Sayed (soft-copy), solve the following problems (pp. 332-333):
(i) Pbm. 6.1 on AR process
(ii)Pbm. 6.3
(iii) Pbms. 6.4 & 6.5 (Price's Thm.), and
(iv) Pbm. 6.6 (Bussgang Thm.).

(v) Consider the update equation for the sign-error LMS algorithm as given below

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \mu \, \text{csgn}[e^*(n)]\mathbf{u}(n)$$

Using the result of Pbm. 6.4 and 6.5 from above, and assuming **u**(n) is Gaussian and e(n) and v(n) (the noise in the data model for d(n)) are jointly Gaussian, show that for the complex case, for step-size sufficiently small the excess MSE can be approximated by

$$\varsigma^{sign-errorLMS} = \frac{\alpha}{2}(\alpha + \sqrt{\alpha^2 + 4\sigma_v^2}), \text{ where } \alpha = \sqrt{\frac{\pi}{4}}\mu \, \text{Tr}(\mathbf{R}_u)$$