Department of Electrical Engineering

EE-5040 : Adaptive Signal Processing

Mar. 2010 **Simulation Assignment #1** KG/IITM

1. *Expectation and Egodicity:* For the 3 difference equations given below, estimate the MxM autocorrelation matrix $\mathbf{R}_{uu}$ for both M=2 and M=3 as follows:

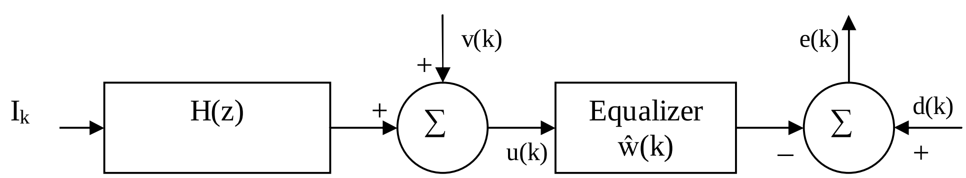
- (i) Calculate the statistical $\mathbf{R}_{uu}$
- (ii) Assuming ergodicity, numerically estimate $\mathbf{R}_{uu}$. Consider time averages obtained using 10, 100 and 1000 sample-functions (Monte-Carlo trials). *Hint*: for AR process, to get WSS, drop the first 500-1000 data samples.

Here, v(n) is AWGN with variance as specified below.

- (a) $u(n) = \cos(2\pi n/50) + v(n)$; $\sigma_v^2=0.1$
- (b) $u(n) = 0.8\, v(n) - 0.2\, v(n-1)$; $\sigma_v^2=1$
- (c) $u(n) = 0.9\, u(n-1) + v(n)$; $\sigma_v^2=0.5$

Bonus question: Set up the Yule-Walker equations (read up from Haykin) for the 3 cases, and verify that $\mathbf{w}=\mathbf{R}_{uu}^{-1}\mathbf{r}$ makes sense

2. *Wiener & Least Squares filtering:* In the figure below, the input $\{I_k\}$ is i.i.d. with $E[I_k^2]=1$, following a uniform pdf (i.e., $P(I_k=+1)=1/2=P(I_k=-1)$), and the noise v(k) is AWGN with variance σ_v^2



The bipolar symbols $\{I_k\}$ suffer inter-symbol interference (ISI) from the channel $H(z) = 1 - z^{-1} + 0.5\, z^{-2}$, and this ISI is compensated by a M^{th} order equalizer, $\hat{\mathbf{w}}$ (or $\hat{\mathbf{w}}(k)$), defined using the LMSE criterion (Wiener filter), or, using a Least Squares criterion. The desired signal is given by d(k), where $d(k)=I_{k-\Delta}$, with $\Delta \geq 0$ being an integer.

- (a) Write a program to calculate the Wiener-Hopf solution $\hat{\mathbf{w}}=\mathbf{R}^{-1}\mathbf{p}$ which should work for any M and Δ. (Assume H(z) and σ_v^2 are known). For the given H(z), the program should be able to calculate $\hat{\mathbf{w}}$ and J_{min} for any σ_v^2,M &Δ.
- (b) To study the effect of window length on the "quality" of R, simulate random data $\{I_k\}$ and $\{v(k)\}$, k=1…N, and find the LS estimate $\hat{\mathbf{w}}_{LS}(n)$ when Δ=0, $\sigma_v^2=0.05$, M=7, and calculate the least sum of error squares J_{min}^{LS} for the following values of N: (i) N=10, (ii)N=100, (iii)N=200, and (iv)N=2000.
- (c) With N=1000, for each of the below situations, find J_{min} and J_{min}^{LS} and tabulate them. Comment on your results.

M	Δ	$\boldsymbol{\sigma_v^2}$	$\mathbf{J_{min}}$	$\mathbf{J_{min}^{LS}}$
3	1	0.01		
10	1	0.01		
15	1	0.01		
10	2	0.01		
10	4	0.01		
10	8	0.01		
10	3	0.01		
10	3	0.05		
10	3	0.10		

Hint: For J_{min}^{ls} get $\hat{\mathbf{w}}_{LS}$ and substitute this in $\sigma_d^2-\mathbf{p}^T\hat{\mathbf{w}}_{LS}$