Department of Electrical Engineering, Indian Institute of Technology, Madras

EE 4190: Digital Communications

November 8, 2012 **Tutorial 3** KG/IITM

1. Recall that the exact expression for average symbol error probability P_e was derived for the orthogonal as well as bi-orthogonal signal sets in class.
(a) Using the Union bound argument, get upper-bound on Pe for any *M*-ary set for both of the above signals.
(b) For the case of the bi-orthogonal set, if the nearest neighbors are only considered in the Union bound, how will be expression be modified?

2. Solve the following problems for "Digital Communications 4^{th} Ed." J.G.Proakis McGraw Hill 2001. Chapter 6 on Carrier and Symbol Synchronisation pp.372 onwards: Pbms. 6.3*, 6.4*, 6.7, 6.8, 6.9*, 6.12*, 6.13, and 6.14. The questions marked with a "*" may require some extra reading from you, since this material was not covered in the course.

3. An uniform iid sequence $I(n) \in \{-1,+1\}$ is transmitted through a FIR channel $H(z) = 1.5 - z^{-1} + 0.5z^{-2}$ and the resultant output *x(n)* is corrupted by an AWGN sequence *u(n)* with variance $\sigma_u^2 = 0.2$. It is required to define a 2-tap linear equalizer to filter the measurement samples *z(n) = x(n)+u(n)*. Assume that {*I(n)*} and {*u(n)*} are mutually uncorrelated.

- **(a)** If the desired sequence is defined by *d(n)= I(n)*, find the 2-tap linear MMSE equalizer for this model. Specify the auto-correlation matrix, the cross-correlation vector, and the equalizer coefficients clearly.
- **(b)** What is the Jmin for this LE-MMSE?
- **(c)** What is the variance of the residual ISI contribution for this LE-MMSE?
- **(d)** Repeat (a), (b), and (c) for the following choices of delay: (a) *d(n)= I(n*-1*)*, and (b) *d(n)= I(n*-2*)*.
- **(e)** Instead, it is required to define a 2-tap Zero Forcing Equaliser for the same model. Specify the coefficients of the LE-ZF clearly.
- **(f)** What is the variance of the residual ISI contribution for this LE-ZF? How does this compare to your answer in part (c)? Comment.

4. Repeat the above problem for a 3-tap linear estimator (i.e., *N*=3). How do your answers compare?

5. An iid 4-PAM sequence $I(n) \in \{-3,-1,+1,+3\}$ goes thro channel $H(z) = 0.5 + z^{-1} - 0.8z^{-3}$ and the resultant output is corrupted by a coloured sequence *u(n)* which is obtained by an AWGN sequence *v(n)* with variance $\sigma_v^2 = 0.3$ passing through a filter $G(z) = 0.5/(1 - 0.8z^{-1})$. It is required to define a MMSE based Decision Feedback Equaliser with 3-taps for the feed-forward section and 2-taps for the feedback section. Assume that {*I(n)*} and {*v(n)*} are mutually uncorrelated. If the desired symbol is *d(n)= I(n-2)*, set up the Weiner-Hopf equations for this measurement model. Clearly provide the entries of the auto-correlation matrix and the cross-correlation vector. (It is not required to determine the coefficients of the DFE.)

6. A 4-tap ISI channel has complex channel coefficients, and QPSK signal are transmitted through it. It is desired to use a Viterbi equalizer at the receiver.

(a) How many states should this VA have?
(b) Sketch a single-stage of the trellis, clearly labeling all the states and showing the transitions.

7. Consider a received signal $z(n) = \sum_{l=0}^{2} f_l \, I(n-l) + v(n)$, where the FIR channel coefficients f_0= -0.4, f_1= 1.0, and f_2= -0.6, and data *I(n)* and noise *v(n)* are mutually uncorrelated with $I(n) \in \{-1,+1\}$ and the noise is AWGN with variance σ_v^2. The Viterbi Algorithm (VA) is to be used to implement MLSE for this measurement model.

- **(a)** Draw a single-stage of the VA, clearly labeling the nodes, and the branches.
- **(b)** The first 4 values of *z(n)* are given as follows: z(1) = -0.6; z(2) = 1.4; z(3) = 0.5; z(4) = -1.7. Assuming that $I(n) = -1,\ n \le 0$, compute the evolution of the VA over the 4 time-intervals. Indicate the values of the Cumulative Metrics (of all the nodes) at the end of time *n*=4.
- **(c)** What is the ML sequence as indicated by the VA at the end of time n=4? (*Hint*: Pick the sequence corresponding to the smallest CM.)
