EE-4190 Digital Communications

Aug. 2012 **Tutorial #1** KG / IITM

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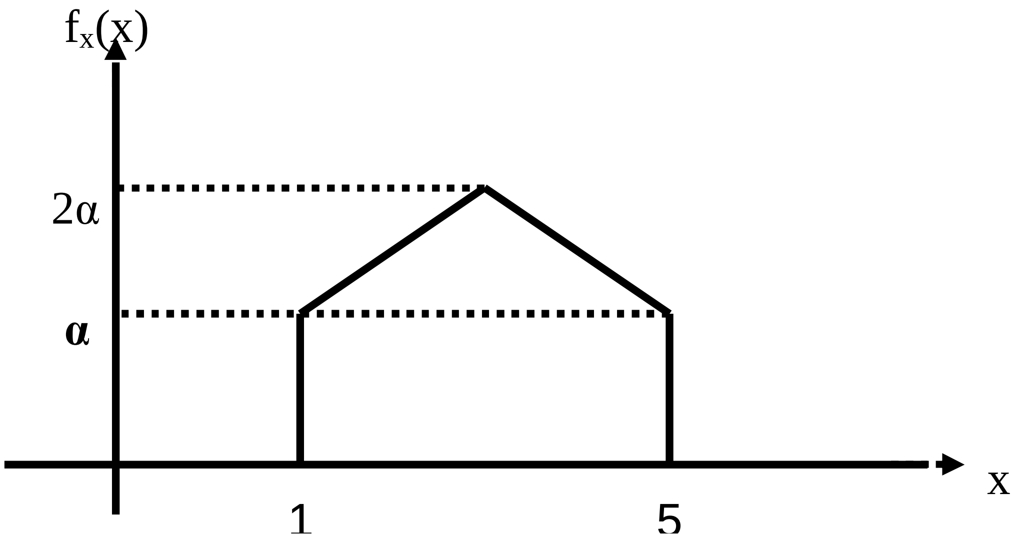
1) A RV X has a PDF as below.



(a) Find the value of ' α ' so that $f_X(x)$ is a valid PDF.

(b) Find $P[(X-2m_X)>2\sigma_X]$ where m_X is the mean and σ_X is the standard deviation of the RV X. Hint: Look before you leap.

(c) Find the upper bound on the above probability using the Chebyshev inequality.

2) Let U and V be i.i.d random variables with uniform pdf between (0,1). A new random variable W is formed as W = U – V. Make a neat labeled sketch of the pdf $f_W(w)$.

3) Consider a normal stochastic process X(t) at 2 time instants t1 and t2 yielding values X1 and X2. If X(t) is characterized by $m_X(t), var_X(t) \, and \, C_X(t,t')$, show that $f({}^{X_2}/_{X_1})$ is also normal with the characteristics:

$$m_{X_2/X_1} = m_2 + r_{1,2}\,\frac{\sigma_{X_2}}{\sigma_{X_1}}(x_1 - m_{X1})$$

$\sigma_{X2/X1} = \sigma_{X2}\sqrt{1 - r_{1,2}^2}$

where $r_{1,2} = \frac{C_X(t,t')}{\sigma_{X_1}\sigma_{X_2}}$.

[All symbols have their usual meanings. Keep the results to memory as they are useful later.]

4) Consider the measurement model X= Θ+W, where W is Gaussian with zero mean and variance σ^2 while Θ is a discrete RV with pdf $f_\Theta(\Theta)$. The conditional mean of Θ is given by $E[\Theta/X] = E[\Theta/X] = 1/h(x)\int g(x,\Theta)\, f_\Theta(\Theta)d\Theta$.

a) Find the correct expressions for h(x) and g(X, Θ) in the above

b) If $\sigma_w^2 = 1$ and $f_\Theta(\Theta) = \frac{1}{2}\delta(\Theta - 1) + \frac{1}{2}\delta(\Theta + 1)$, find E[Θ/X=1.5].

5) In a 2x1 real random vector $\overline{X} = [X_1\ X_2]^T$, the 1^{st} and 2^{nd} order moments are given by $E[X_i^2] = 6$, $E[X_i X_j] = 3$, and $E[X_i] = 1$.

a) What is the 2x2 covariance matrix K_X?

Find a whitening transformation D such that $\overline{Y} = D\overline{X}$ has a covariance matrix $K_Y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. [Note that $E[\overline{Y}] \neq 0$]

6) Consider the following equation

$L_t\{X(t)\} = L_t^{(0)}\{X(t) + Y(t)$ where

L_t *is a non homogeneous linear transformation*

$L_t^{(0)}$ *is a homogeneous linear transformation*

$Y(t)$ *is a non − random function*

Now if $Y(t) = L_t\{X(t)\}$, show that

i) $m_Y(t) = L_t\{m_X(t)\}$

ii) $C_{Y(t,t')} = L_t^{(0)}\{\, L_t^{(0)}\{C_{X(t,t')}\}\}$ where C_X, C_Y represent the covariance functions of $X(t)$ and $Y(t)$ respectively.

iii) $R_{XY}(t,t') = L_{t'}^{(0)}\{R_X(t,t')\}$

7) If X(t) is a stochastic process characterized by $m_X(t)$ and $C\_X(t,t')$, compute $m_Y(t),\ C_Y(t,t'),\ var_Y(t)\ and\ C_{XY}(t,t')$ if

a) $Y(t) = {}^{dx(t)}/_{dt}$

b) $Y(t) = \int_0^t X(t)\,dt$

8) The following four waveforms are used for signaling in a digital communication system:
(rect(t) = 1 for 0 _ t < 1 and zero elsewhere)

$s_0(t) = rect(t) + rect(t - 2)$
$s_1(t) = rect(t - 1) + rect(t - 3)$
$s_2(t) = rect(t - 1) + rect(t - 2)$
$s_3(t) = rect(t - 1) - rect(t - 3)$

(i) Determine an orthonormal basis and the corresponding constellation in two ways:
(a) by using Gram-Schmidt Orthogonalisation starting with $s_0(t)$ and going in sequence, and
(b) by inspection of the waveforms without any computations.
(ii) Verify that one constellation can be obtained from the other simply by rotation.
(iii) What is the distance of each point si from the origin? Is it the same in both constellations? Why? How do the distances between s_0, s_1, s_2 and s_3 compare in either case?

9) Problem-2.24 and 2.25 from "*Fundamentals of Digital Communications*", by Upamanyu Madhow.