Department of Electrical Engineering,
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EE 4140: Digital Communications

October 2016 **Tutorial 3** KG/IITM

1. Solve the following problems from the 8th chapter in the text-book (Proakis and Salehi), starting with page. 561 in the E-version. The problems marked with "*" are a little bit harder, since they were not discussed in the class (as yet).

- ➔ Problems **8.1, 8.2,** and **8.3*.**
- ➔ All problems from **8.9 to 8.22**. The possibly difficult problems in this set are 8.17* and 8.19*.

2. In the figure below, the input $\{I_k\}$ is i.i.d with $E[I_k^2]=1$ and $P[I_k=+1]=1/2= P[I_k=-1]$. The AWGN v(k) has variance $\sigma_v^2=0.2$ and $E[I_k\, v(i)]=0$ for all k,i.

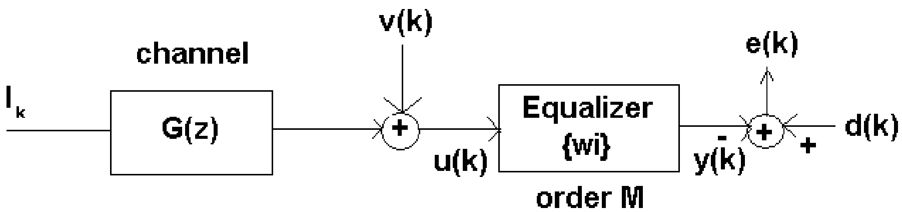


Figure 1 – FIR equalization

The channel $G(z)=1-z^{-1}+0.5\, z^{-2}$ and the equalizer has an order equal to M. For the following situations, compute manually $\mathbf{R}$, $\mathbf{p}$, and eventually $\mathbf{w}_{opt}=\mathbf{R}^{-1}\mathbf{p}$.

a) M=2, d(k)=I(k)
b) M=2, d(k)=I(k-1)
c) M=2, d(k)=I(k-2)
d) What is the J_{min} in each of the above cases?
e) Now, consider a Decision Feedback Equaliser (DFE) with M=2 taps in the feedforward section and 2 taps in the feedback section. For each of the cases in (a), (b), and (c), what will be the resultant **R** and **p** in this case?

3. A real WSS process u[n] is to be filtered by a 2-tap estimator **w** such that $E\{|y[n]|^2\}$ is minimized where $y[n]=\mathbf{w}^T\mathbf{u}[n]$, subject to $\mathbf{w}^T\mathbf{g}=1$ where $\mathbf{g}=[1\ -1]^T$.

a) Given that r[0]=5, and r[1]=2, use the Lagrange multiplier technique to determine **w**. What is the value of the Lagrangian (constant)?
b) Repeat part (a) with the alternative constraint $\mathbf{w}^T\mathbf{w}=1$. Specify all values of **w**.

4. A uniform real i.i.d sequence {d[k]} with $E\{|d[k]|^2\}=1$ is filtered by $H(z)= 1-0.5z^{-1}+(1/3)\, z^{-3}$ and the resulting output is corrupted by a coloured noise which is a result of AWGN filtered by $1+0.8z^{-1}$ to give the measurements {u[k]} where the variance of the AWGN is 0.25 and this AWGN is uncorrelated with {d[k]}.

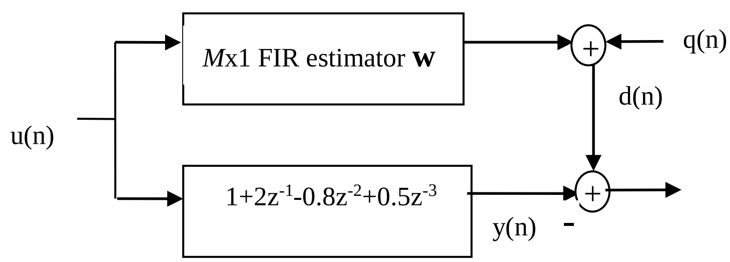
a) Find $\mathbf{R}_{uu}$ of size 3×3.

b) Find a 3×1 $\mathbf{p}=E\{\mathbf{u}[k]d[k-\Delta]\}$ for (i)$\Delta=1$,(ii)$\Delta=4$.

5. In the system identification model shown below, a M^{th} order FIR structure is used to define the estimator **w**.

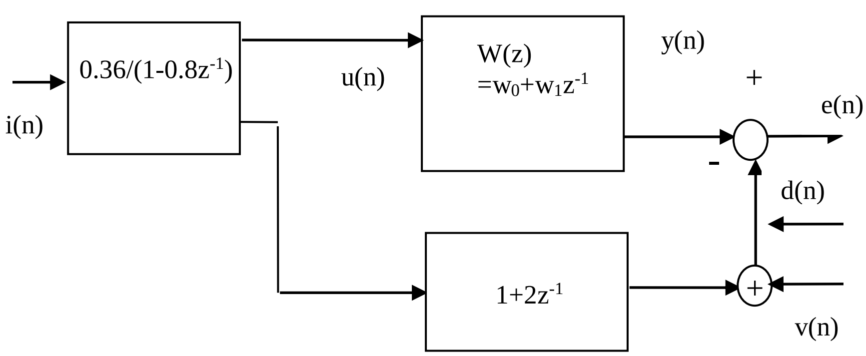
(a) Given that the signal sequence u(n) and the noise sequence q(n) and self and mutually uncorrelated with variance $\sigma_u^2=1$, and $\sigma_q^2=0.1$, respectively, use the MMSE formulation to determine **w** and J_{min} for the following choices of *M*: (i) *M*=2; (ii) *M*=3; (iii) *M*=4; (iv) *M*=6

(b) Based on your above answer, can you make a rough plot of Jmin(*M*) versus *M* for a suitable range of vales for *M*?



6. Referring back to Figure 1 in Pbm. 1, the input $\{I_k\}$ is i.i.d with $E[I_k^2]=1$ and $P[I_k=+1]=1/2= P[I_k=-1]$. The AWGN v(k) has variance $\sigma_v^2=1$ and $E[I_k\, v(i)]=0$ for all k,I, and the channel response $G(z)=1/(1-0.8z^{-1})$. If **w** is of order M=2, find $\mathbf{w_o}$ for (a) $\sigma_v^2=1$, (b) $\sigma_v^2=0.3$ and (c) $\sigma_v^2=0$.

7. Consider the measurement model below, where a linear MSE (Wiener) estimator is to be defined for the equalizer W(z) which will minimize $E[e^2(n)]$. Here, input i(n) is white noise with unit variance, and additive noise v(n) has variance $\sigma_v^2 = 0.10$ and is uncorrelated with i(n).

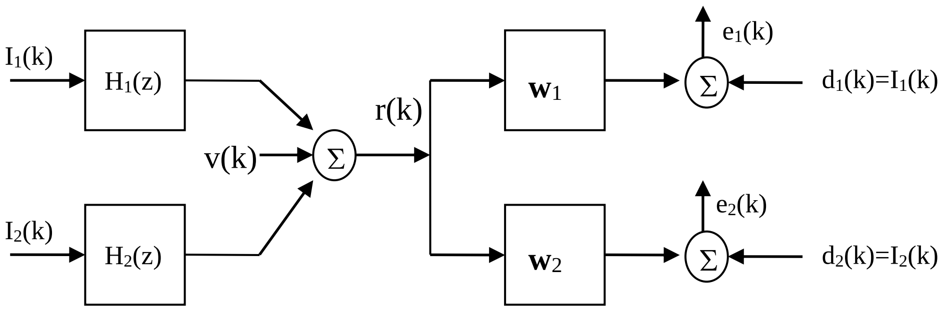


(a) Find the correlation matrix **R** and the cross-correlation vector **p**.

(b) What is the LMSE estimate for W(z)?

(c) What is the Jmin for this LMSE problem?

8. Two mutually uncorrelated i.i.d bipolar bit-streams $I_1(k)$ and $I_2(k)$ with variances σ_1^2 and σ_2^2 are filtered by $H_1(z)=0.8+z^{-1}$ and $H_2(z)=1-0.7z^{-1}$, respectively. Their filtered outputs get added to an AWGN process v(k) with variance σ_v^2, so as the produce the measurements r(k) as shown in the figure below. The receiver uses two equalizers, $\mathbf{w}_1$ and $\mathbf{w}_2$, to recover (estimate) $I_1(k)$ and $I_2(k)$, respectively.



(a) If $\sigma_1^2 = 3$, and $\sigma_2^2 = \sigma_v^2 = 0$, specify the MMSE estimate of $\mathbf{w}_1$ of order M=5. *Hint*: At infinite (sufficiently high) SNR, MMSE and zero-forcing estimators are identical.

(b) Now if $\sigma_1^2 = 1, \sigma_2^2 = 0.8$, and $\sigma_v^2 = 0.2$, which of the above two equalizers will give a better estimate of the corresponding data symbol? *Hint*: That equalizer providing the lower J_{min} will be the better one.