Department of Electrical Engineering,
Indian Institute of Technology, Madras

EE 4140: Digital Communications

September 2016 **Tutorial 2** KG/IITM

1. Find the compact ortho-normal basis set, and using it, make a clear labeled plot of the signal constellation for the signal set shown in Fig-1.

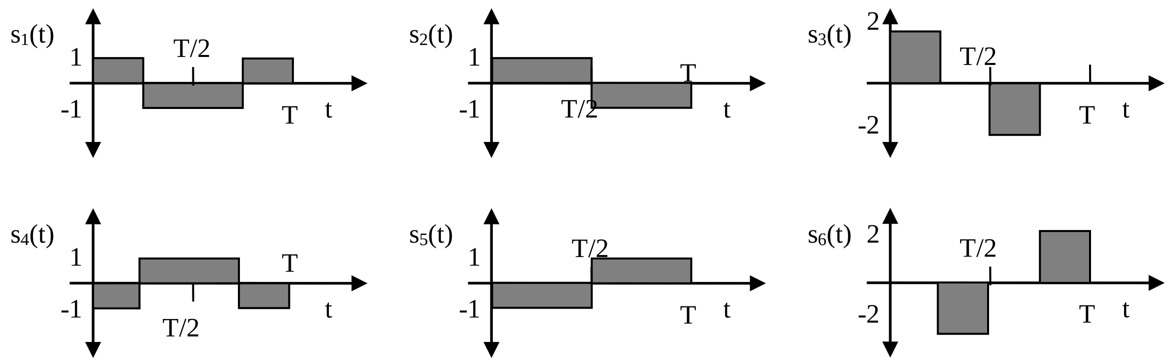


Fig-1

2. The signal g(t) is sent through a channel with impulse response h(t), where the two functions are shown in Fig-2. Make a labeled plot of the ideal matched filter's impulse response. *Hint:* Assume single-shot communication.

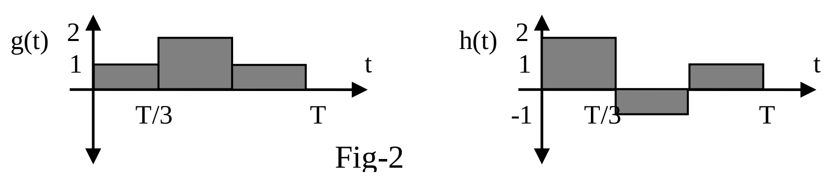


Fig-2

3. Find the autocorrelation function S(t) for the given signal g(t) in Fig3. Draw the signal S(t).

where

$$S(t) = \int_{-\infty}^{\infty} g(\tau) g(t-\tau) d\tau$$

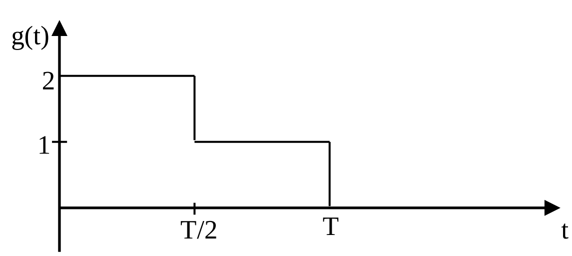


Fig-3

4. Find the minimum distance and multiplicity for given signals in Fig-4.

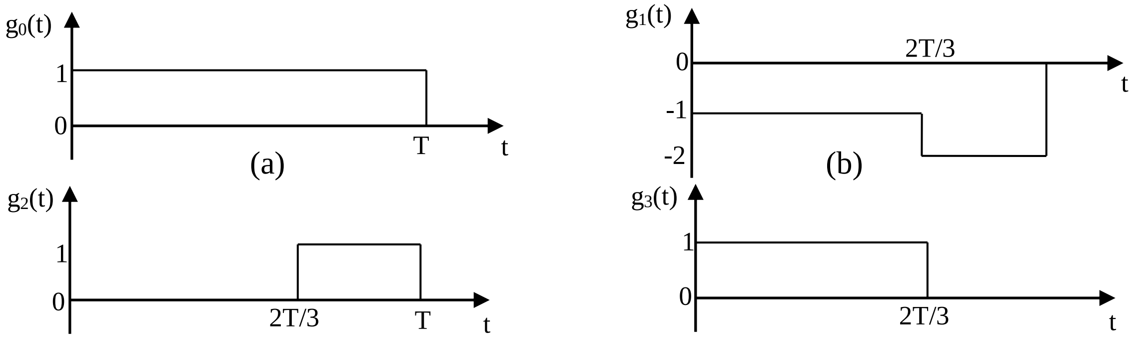


Fig-4

5. Find out the compact basis function for signals given in Fig-5.

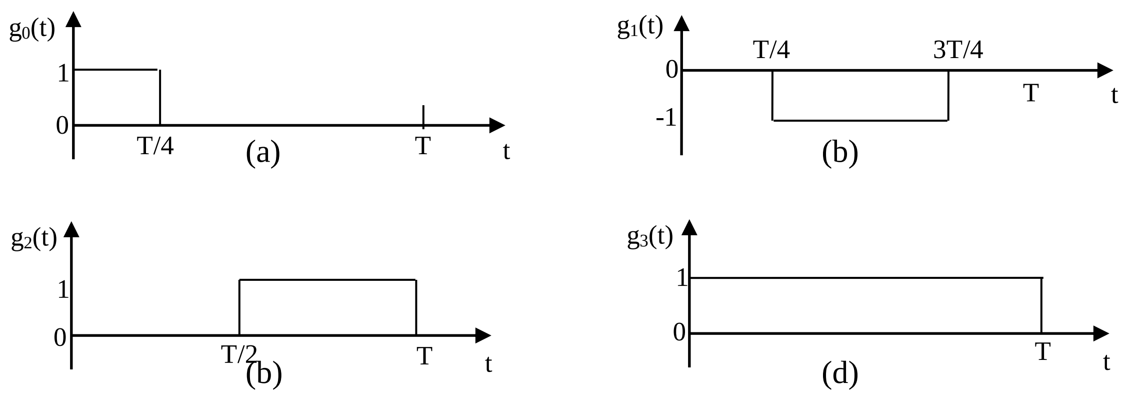


Fig-5

6. Consider a band-pass signal $s(t) = I(k)g(t)Cos(2\pi f_c t)$ for $kT \le t \le (k+1)T$ where the pulse shape $g(t)$=sqrt(2/T) for 0≤t≤T. Here, message symbol $I(k) \in \{+3d, +d, -d, -3d\}$, and the received sample at the output of the matched filter can be written as $z(k) = \alpha\, I(k) + v(k)$ where $v(k)$ is WGN with variance No/2, and the real scalar α accounts for any possible gain(scaling) error encountered in the AGC-ADC operations.

(a) If the average energy E_a for this signal set is 4 Joules, what is d? *Hint:* Also, relate this E_a to the distance 2d between the neighbouring points in the constellation in order to answer part-(c).

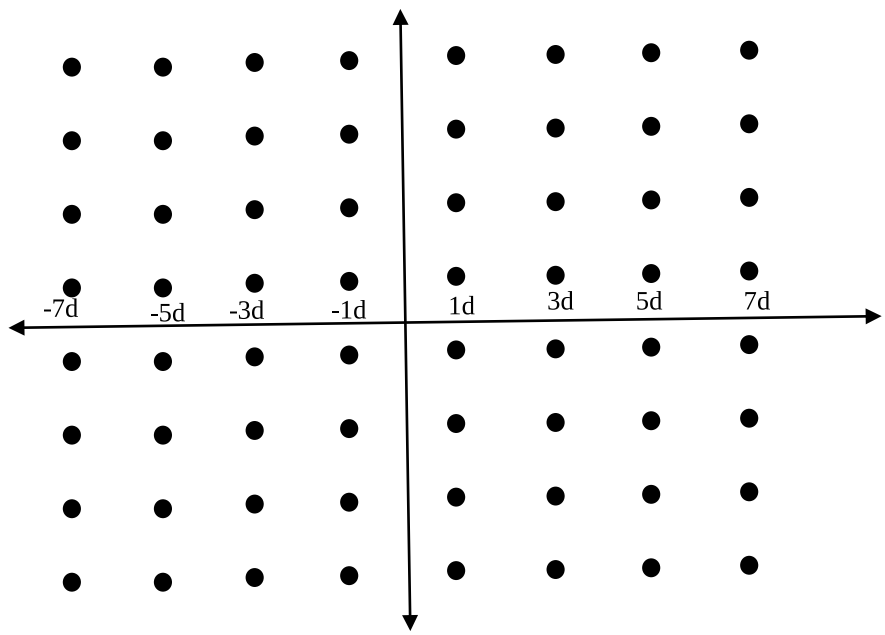
(b) For α=1, find the exact expression for the average probability of symbol error P_e in the above AWGN channel. Express your answer in terms of $q(d) = Q\left(\frac{d}{\sqrt{N_0/2}}\right)$ with 2d as in part (a).

(c) Now, if α=1.5 and this knowledge is not known at the receiver, find the new expression for P_e when the same decoder (decision regions) as in part (b) is used.

7. Derive the average probability of symbol error P(e) for the square 64-QAM constellation shown below in terms of $q(d)$.

(a) Assume instead that the union bound is used only on the "nearest neighbor" symbols to compute the bound on $P_{UB}(e)$. What is this expression?

(b) How does this compare with the true P(e)? Numerically evaluate both of them for Eb/No = 10dB.



8. Consider a band-pass signal $s(t)=I_1(k)g(t)Cos(2\pi f_c t)+I_2(k)g(t)Sin(2\pi f_c t)$, for $kT \le t \le (k+1)T$, where the pulse shape g(t)=sqrt(2/T) for $0 \le t \le T$. If $I_1(k) \in \{+1,-1\}$, while $I_2(k) \in \{+3, +1, -1, -3\}$, determine the following:

(d) What is the ortho-normal basis set and plot the corresponding signal constellation.

(e) What is the average energy E_a for this signal set? *Hint:* Also, relate this E_a to the distance 2d between the neighbouring points in the constellation in order to answer part-(c).

(f) Find the exact expression for the probability of symbol error in an AWGN channel with PSD No/2. Express your answer in terms of q where q(d)=Q(d/sqrt(No/2)) with 2d as in part (b).

(g) Perform Gray coding for the constellation. Using this, provide the expression for the bit error probability (i.e., bit error rate) for the above measurement model.

9. Do relevant problems from the text book.