Department of Electrical Engineering – IIT Madras

EE4140 – Digital Communication Systems

Tutorial 1 August 2016

1. Let X and Y be independent and identically distributed random variables with common density function f and cumulative distribution function F. Find the density and distribution functions of $U = \max(X, Y)$ and $V = \min(X, Y)$.

2. Let X be an $N(0, \sigma^2)$ random variable. The random variable Y is given by $Y = g(X)$. Find the density function of Y for each of the following choices of the function g:
   a. $g(x) = e^x$
   b. $g(x) = x^2$
   c. $g(x) = \begin{cases} x, & x \geq 0 \\ 0, & x < 0 \end{cases}$
   d. $g(x) = |x|$
   e. $g(x) = \begin{cases} -b, & x \leq -b \\ b, & x \geq b \\ x, & |x| < b \end{cases}$

3. Let X and Y be independent random variables having the exponential distribution with parameters α and β respectively. Find the density function of $Z = X + Y$. (Note : The density function f of an exponentially distributed random variable with parameter λ is $f(x) = \lambda e^{-\lambda x}$

4. Find the density function of $Z = X + Y$ when X and Y have the joint density function
$$f(x, y) = 0.5(x + y)e^{-(x+y)}; x, y \geq 0$$

5. Let X and Y be jointly Gaussian distributed random variables with zero means, unit variances and correlation ρ. Find the joint and marginal density functions of $U = X + Y$ and $V = X - Y$. (Hint : U and V are jointly Gaussian distributed as well)

6. Let X and N be independent Gaussian random vectors, having means μ_x, μ_y and covariances $\sum_x, \sum_y$ respectively. Find the mean and covariance matrix of the random vector $Z = HX + N$, where H is a constant matrix of appropriate dimensions

7. X and Y are random variables following a bivariate normal distribution. Both X and Y have mean μ, variance σ^2, and their correlation coefficient is ρ. Find the conditional mean $E(Y|X = x)$ and the conditional variance $var(Y|X = x)$