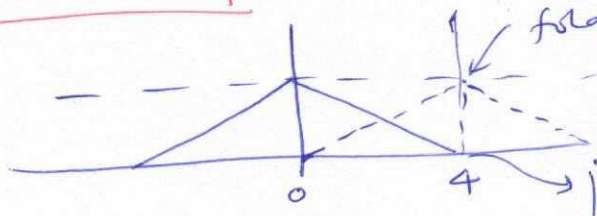
1. 5 marks

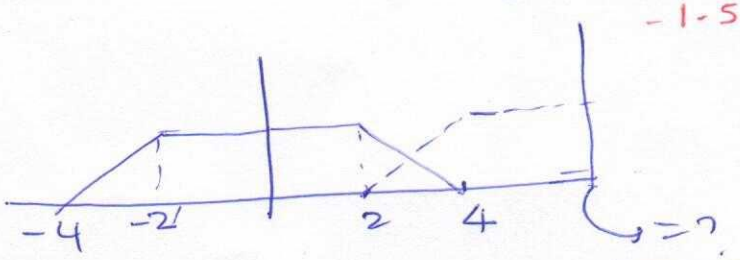
(a)



$\frac{1}{T} = 4\ MHz$

-1.5-

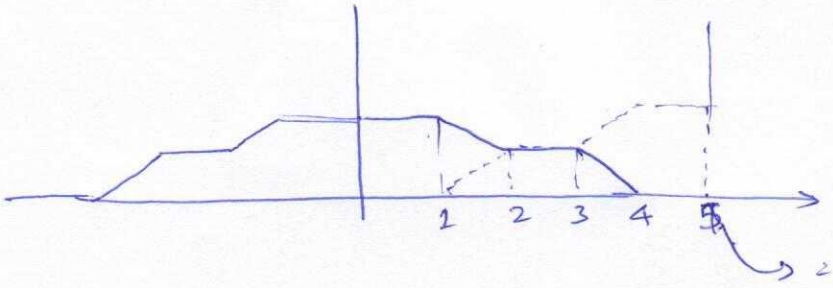
(b)



$\frac{1}{T} = 6\ MHz$

-1.5-

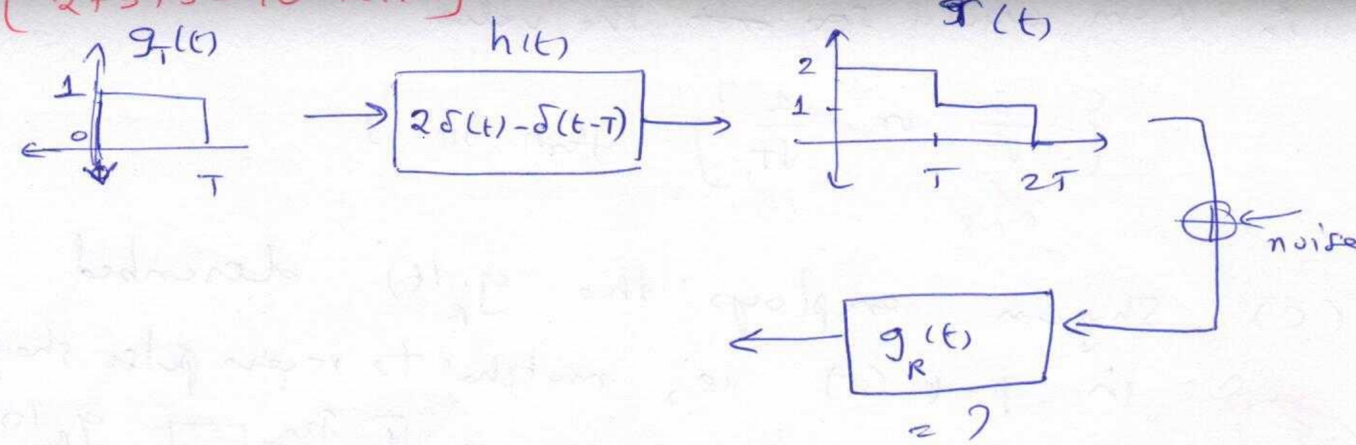
(c)



$\frac{1}{T} = 5\ MHz$

-2-

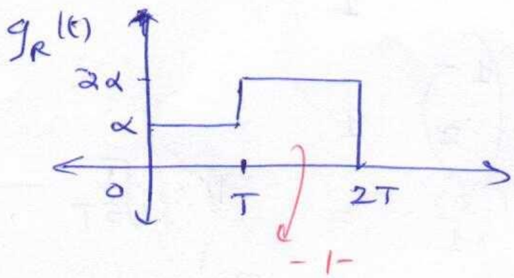
2. [2+3+5 = 10 marks]



(a) Best possible $g_R(t)$ for "single-shot" communications

(-2-) is $g_R(t) = r(2T - t)$, after normalisation.

i.e.



& $\alpha^2 T + 4\alpha^2 T = 1$ ← unit gain

$\Rightarrow \alpha^2 = \frac{1}{5T} \Rightarrow$ $\boxed{\alpha = \sqrt{\frac{1}{5T}}}$;

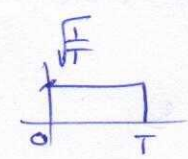
-1-

(b) continuous signalling & Rx using $g_R(t) = \frac{1}{\sqrt{T}} g_T(T-t)$

(-3-) Let the k^{th} bit transmitted = +1

Since memory is only 1 symbol long (i.e., length 2T),
we need to consider only k-1 & k+1 bits to see
the impact of ISI. only 4 combinations of $(I(k-1), I(k+1))$
to be considered

$I(k-1)$ $I(k)$ $I(k+1)$



Pattern 111 → 1   1   1

2 1
2   1   2   1
↓   ↓
3   3 → This is the value "seen" after MF @ time "kT"
(Actually $\frac{3}{\sqrt{T}}$

Pattern -1 1 1 → -1   1   1

2 1
-2   -1   2   1 → value seen is $\frac{1}{\sqrt{T}}$
↓
1

Pattern -1 1 -1 → $1/\sqrt{T}$

2 1

$\therefore$ Ram would see ~~only~~ the values

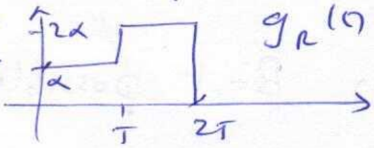
$$\left\{ \frac{3}{\sqrt{T}} \text{ or } \frac{1}{\sqrt{T}} \right\} \text{ for bit "1"}$$

1.5 — 1.5 —

(c) Shyam employs the $g_R(t)$ described in part (a) i.e., matched to receive pulse shape

-5-

$\alpha = \sqrt{\frac{1}{5T}}$



Pattern	$I(k-1)$	$I(k)$	$I(k+1)$	
1 1 1	1	1	1	
	2	2, 1 → 3 ×2	1, 2 → 3 ×1	1

$3 \times 2 + 3 \times 1 = 9 \times \sqrt{\frac{1}{5T}} = \frac{9}{\sqrt{5}} \frac{1}{\sqrt{T}} = \frac{4.025}{\sqrt{T}}$;

Pattern -1 1 1

$1 \times 2 + 3 \times 1 = 5 \times \sqrt{\frac{1}{5T}}$;

(ii) Pattern -1 1 -1 ;

$\rightarrow 1 \times \sqrt{\frac{1}{5T}}$

&

Pattern 1 1 -1 ;

$= 5 \times \sqrt{\frac{1}{5T}} = \sqrt{5} \times \frac{1}{\sqrt{T}}$

$\sqrt{5} \approx 2.236$

Note: Shyam gets

$$\left\{ \frac{4.025}{\sqrt{T}}, \frac{2.236}{\sqrt{T}}, \& \frac{0.447}{\sqrt{T}} \right\}$$

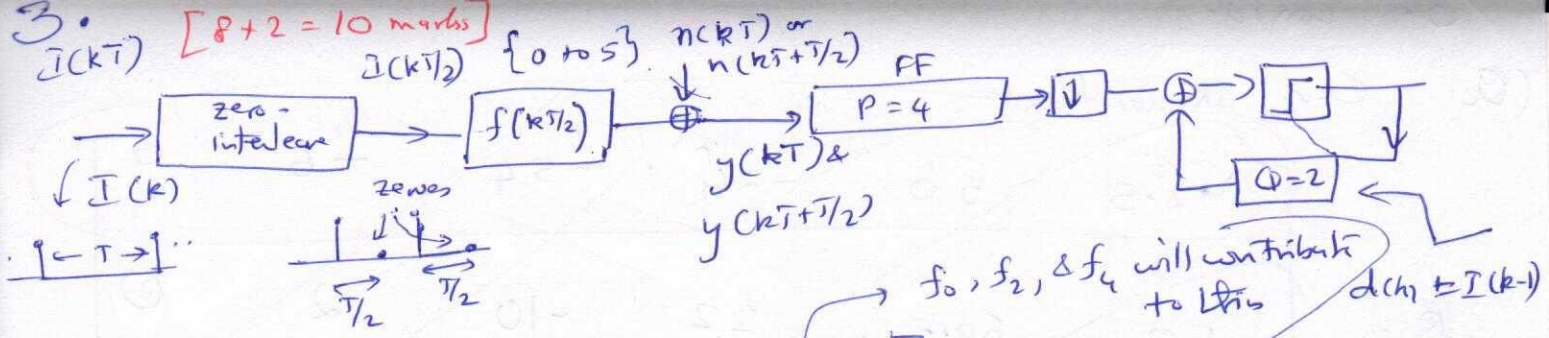
-3-

↑ max is more than Ram's

↑ min is however smaller than

-2-

3. [8+2 = 10 marks]



$f_0, f_2, \& f_4$ will contribute to this

$f_1, f_3, \& f_5$ to this

Measurement Vector $\underset{6\times1}{\bar{u}(kT)} =$

$$\begin{bmatrix} y(kT) \\ y(kT-T/2) \\ y(k-1)T \\ y((k-1)T-T/2) \\ I(k-2) \\ I(k-3) \end{bmatrix}$$

$\sigma_I^2 = 2$;

$\sigma_n^2 = 0.5$;

since, $d(k) = I(k-1)$

-1-

where

$$y(kT) = I(k) + 6I(k-1) - 3I(k-2) + n(kT)$$

$$y(kT-T/2) = -2I(k) + 5I(k-1) + I(k-3) + n(kT-T/2)$$

$$y((k-1)T) = I(k-1) + 6I(k-2) - 3I(k-3) + n((k-1)T)$$

$$y((k-1)T-T/2) = -2I(k-1) + 5I(k-2) + I(k-4) + n((k-1)T-T/2)$$

$$R = E\left[\bar{u}\,\bar{u}^T\right] \leftarrow 6\times6 \text{ matrix}$$

$$\bar{p} = E\left[\underset{\uparrow d(k)}{I(k-1)}\ \bar{u}\right] ;$$

3. contd.

(a) Can show that

$$R = \begin{bmatrix} 92.5 & 50 & -24 & -54 & -6 & 0 \\ 50 & 60.5 & 22 & -10 & 2 & 0 \\ -24 & 22 & 92.5 & 50 & 12 & -6 \\ -54 & -10 & 50 & 60.5 & 10 & 2 \\ -6 & 2 & 12 & 10 & 2 & 0 \\ 0 & 0 & -6 & 2 & 0 & 2 \end{bmatrix}$$

← 4 marks to these

1+

(b) $\bar{p} = E\left[I(k-1)\,\bar{u} \right]$

$$= \begin{bmatrix} 12 \\ 10 \\ 2 \\ -4 \\ 0 \\ 0 \end{bmatrix};$$

← 2 marks for those 4 entries