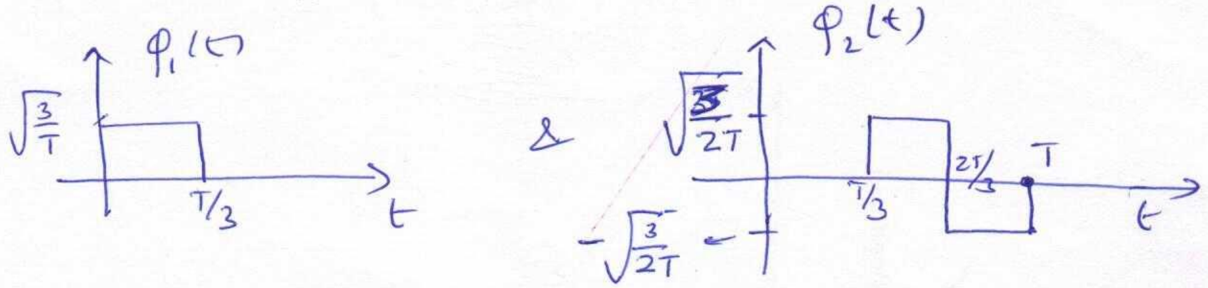
#1

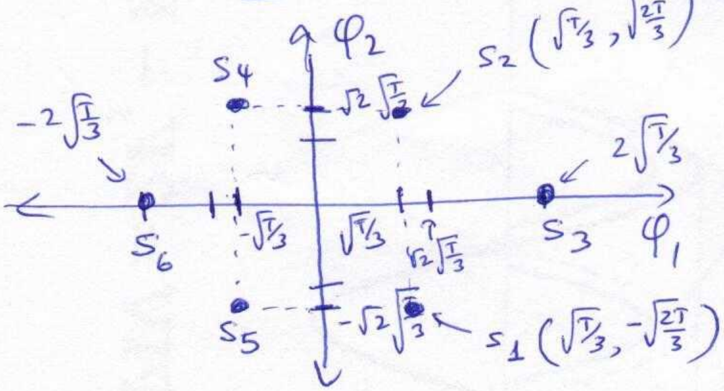
6/6

(a) $\rightarrow$ same for $\boxed{A}$ & $\boxed{B}$



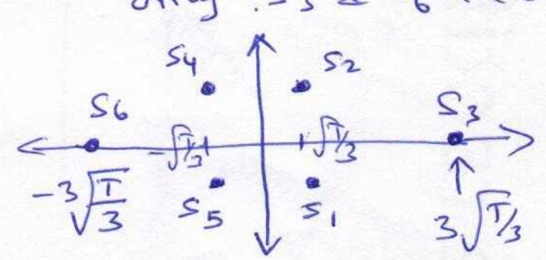
(b) Small difference in signal set (& constellation) between $\boxed{A}$ & $\boxed{B}$

for $\boxed{A}$



for $\boxed{B}$

only s_3 & s_6 are changing

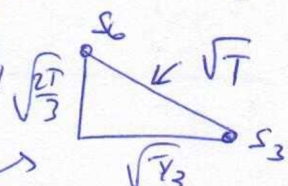


(c) Minimum distance for $\boxed{A}$

$$E_a = \left(\frac{T}{3} + \frac{2T}{3}\right) \times \frac{4}{6} + \frac{4T}{3} \times \frac{2}{6}$$

$$\Rightarrow \boxed{E_a = \frac{10T}{9};}$$

$\Rightarrow d_{min} = \sqrt{T}$ see figure

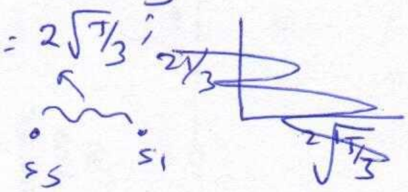


i.e. $$\boxed{d_{min} = 3\sqrt{\frac{E_a}{10}}};$$

Minimum distance for $\boxed{B}$

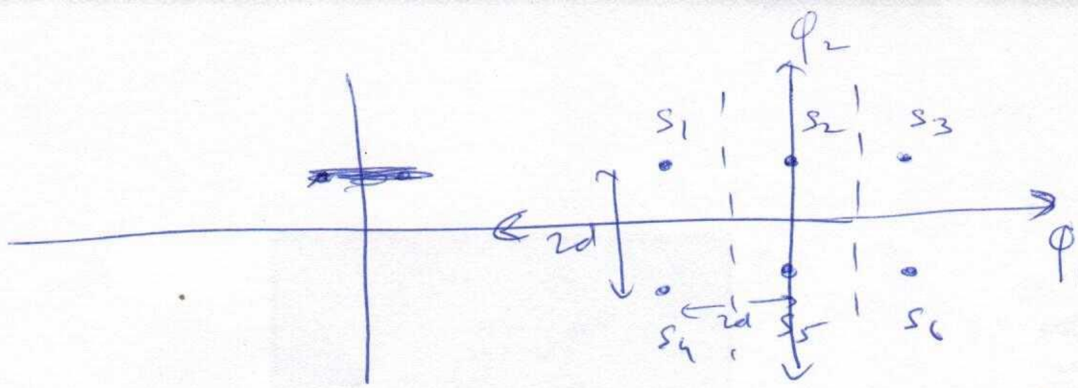
$E_a = \frac{5T}{3}$; and

$d_{min} = 2\sqrt{T/3}$;



$$\Rightarrow \boxed{d_{min} = 2\sqrt{\frac{E_a}{5}}};$$

#2:



$$P_{c_1} = P_{c_4} = P_{c_6} = P_{c_3} = (1-q)^2, \quad q(d) = Q\left(\frac{d}{\sqrt{N_0/2}}\right);$$

$$P_{c_2} = P_{c_5} = (1-2q)(1-q)$$

(a) $\therefore P_e = 1 - \left(\frac{4}{6}(1-q)^2 + \frac{2}{6}(1-2q)(1-q)\right)$

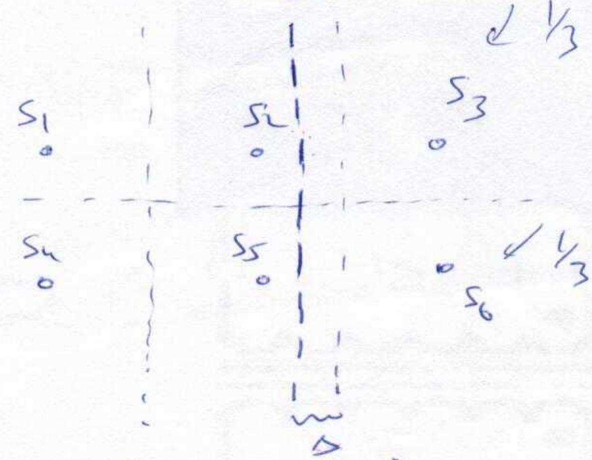
$$= 1 - \left(\frac{4(1+q^2-2q) + (2-2q)(1-q)}{6}\right)$$

$$= 1 - \left(\frac{4+q^2-8q+2-4q-2q+4q^2}{6}\right)$$

$$= \frac{\cancel{6} - \cancel{6} + 14q - 5q^2}{6} = \frac{14q-5q^2}{6};$$

(b)

$\boxed{A}$

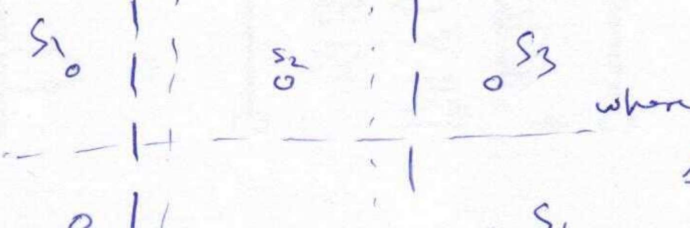


$$\Delta = \frac{N_0/2}{d}\ln\left(\frac{p_1}{p_2}\right)$$

$$\frac{1/3}{1/12} = 4$$

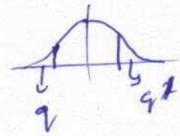
$$\Delta = \frac{\frac{N_0}{2}}{d}(\ln 4)$$

$\boxed{B}$



where $\Delta = \frac{\frac{N_0}{2}}{d}(\ln 4)$

2 contd :



(b)

$\boxed{A}$ $\therefore$ $P_e \Rightarrow P_{c_1} = P_{c_4} := (1-q)^2$

$$P_{c_2} = P_{c_5} = (1-q)(1-q-q_1)$$

$$P_{c_3} = P_{c_6} = (1-q)(1-q_2)$$

$$q_1 = Q\left(\frac{d-\Delta}{\sqrt{N_0/2}}\right);$$

$$\Delta = \frac{N_0/2}{d} \ln \underset{1.3863}{4}$$

$$q_2 = Q\left(\frac{d+\Delta}{\sqrt{N_0/2}}\right); \quad \& \quad q = Q\left(\frac{d}{\sqrt{N_0/2}}\right);$$

(b)

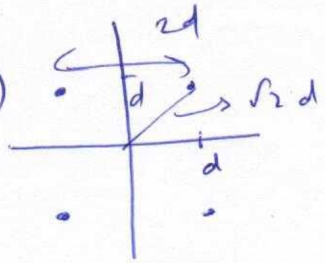
$\boxed{B}$ $P_e \Rightarrow P_{c_1} = P_{c_4} = P_{c_3} = P_{c_6} = (1-q)(1-q_1);$

$$P_{c_2} = P_{c_5} = (1-q)(1-2q_2);$$

with q_1 and q_2 as above;

#3. (same for (A) & (B) papers) 4/6

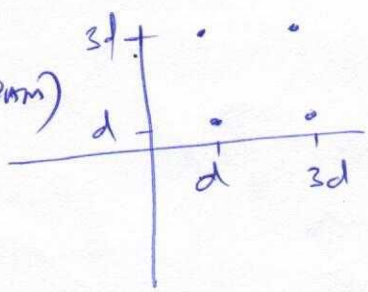
x (QPSK)



$E_{ax} = 2d^2 = \frac{d_x^2}{2}$ & $2E_{bx} = E_{ax}$

$\Rightarrow E_{bx} = \frac{E_{ax}}{2} = \frac{2d^2}{2} = d^2$; $= \frac{d_x^2}{4}$

y (16-QAM)



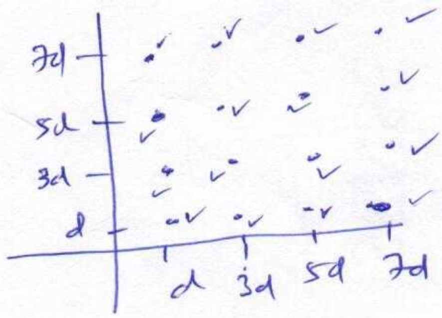
$$E_{ay} = \frac{4}{16}\cdot(d^2+d^2) + \frac{8}{16}(9d^2+d^2) + \frac{4}{16}(9d^2+9d^2)$$

$$= \frac{d^2}{2} + 5d^2 + \frac{9d^2}{2}$$

$\Rightarrow E_{ay} = 10d^2 = \frac{5}{2}d_y^2$ & $4E_{by} = E_{ay}$

$\Rightarrow E_{by} = \frac{E_{ay}}{4} = \frac{10d^2}{4} = \frac{5d^2}{2}$; $= \frac{5}{8}d_y^2$

z



$$E_{az} = \frac{4}{64}(d^2+d^2) + \frac{8}{64}(9d^2+d^2) + \frac{4}{64}(9d^2+9d^2) + \frac{8}{64}(25d^2+d^2) + \frac{8}{64}(25d^2+9d^2) + \frac{4}{64}(25d^2+25d^2) + \frac{8}{64}(d^2+49d^2) + \frac{8}{64}(49d^2+9d^2) + \frac{8}{64}(49d^2+25d^2) + \frac{4}{64}(49d^2+49d^2)$$

$$E_{az} = \frac{1}{64}\Big[8d^2 + 80d^2 + 72d^2 + 208d^2 + 272d^2 + 200d^2 + 400d^2 + 464d^2 + 592d^2 + 392d^2$$

160
480
1600
1200
248
2688

$$E_{az} = \frac{2688d^2}{64} = 42d^2 = \frac{21}{2}d_z^2$$

$$6E_{bz} = E_{az} \Rightarrow E_{bz} = \frac{E_{az}}{6} = \frac{42d^2}{6} = 7d^2 ; = \frac{7}{4}d_z^2$$

#3 contd

All E_b's being equal

$$\Rightarrow \quad 7 d_z^2 = \frac{5 d_4^2}{2} = d_x^2$$

$$\Rightarrow \quad \boxed{d_x = \sqrt{7}\, d_z} \ (= 2.646);$$

Expressing answer wrt d_z,

(c)

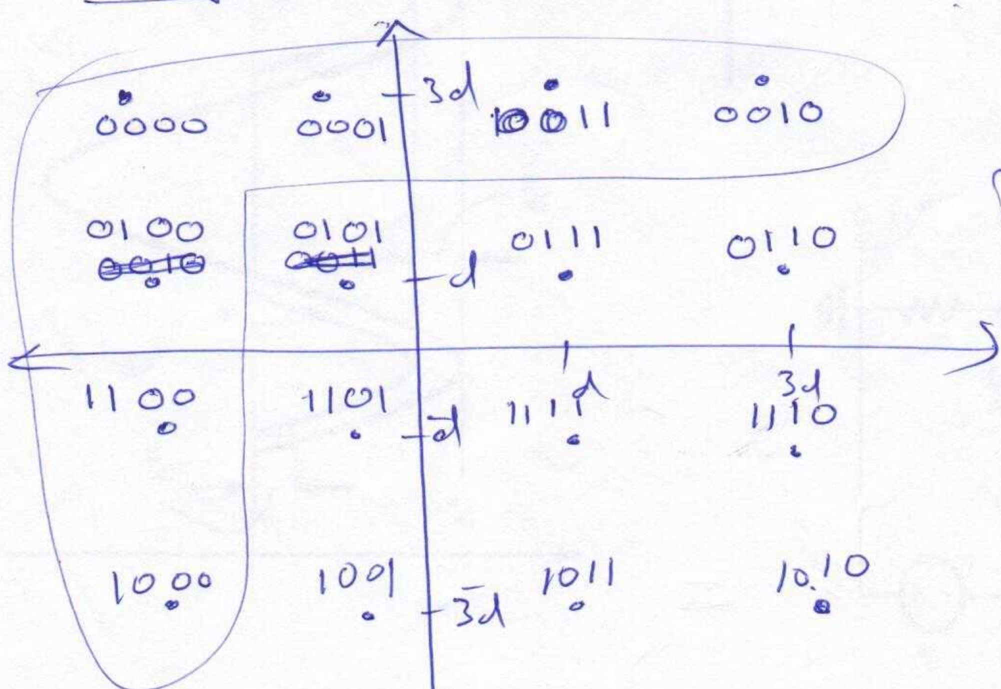
$$\& \quad \boxed{d_4 = \sqrt{\frac{14}{5}}\, d_z} \ (= 1.6733);$$

(a) QPSK → 2 bits/symbol

16-QAM → 4 bits/symbol

64-QAM → 6 bits/symbol

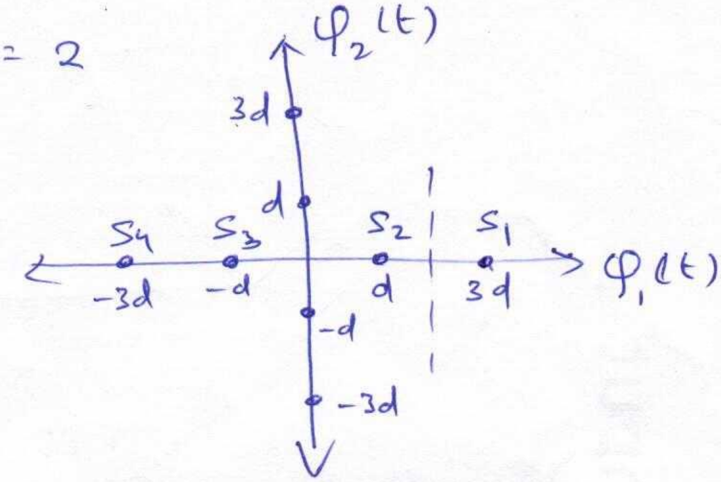
(b) Gray coded 16-QAM



Check

#4. Same for (A) & (B) papers.

(a) Nor $N = 2$



(b) Exact $P(e) \rightarrow$ first get $P_c(s_1) = P_{c1}$
and $P_c(s_2) = P_{c2}$

$$P_{c1} = \int_{2d}^{\infty} f_N(v - 3d)\left(1 - 2\int_{v}^{\infty} f_N(\alpha)\,d\alpha\right) dv$$

&

$$P_{c2} = \int_{0}^{2d} f_N(v - d)\left(1 - 2\int_{v}^{\infty} f_N(\alpha)\,d\alpha\right) dv$$

Note: $P_{c1} > P_{c2}$;

Avg. P of correct symbol detection $P(C) = \frac{1}{2} P_{c1} + \frac{1}{2} P_{c2}$ & $\boxed{P(e) = 1 - P(c)}$

(c) Using Union bound only on Nearest Neighbours (NN)

$P_1 \rightarrow S_1 \rightarrow$ has only 1 NN

$P_2 \rightarrow S_2 \rightarrow$ has 2 NN $\qquad q = Q\left(\frac{d}{\sqrt{N_0/2}}\right)$;

$P_1 = q$; $P_2 = 2q$; || Avg.