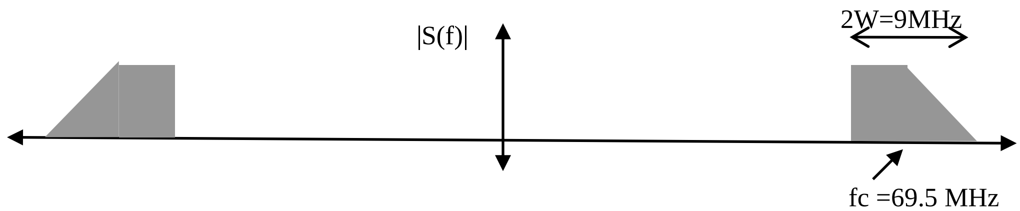
Dept. of Electrical Engineering, IIT Madras
EE 4140: Digital Communication Systems

Aug. 2024 **Tutorial #1** KG / IITM

1. A low-pass signal of one-sided bandwidth of W=1.25MHz is sent as a DSB-SC signal. If the receiver uses an IF sampling scheme, with center frequency f_{IF} = 71MHz, determine the least sampling rate f_S required.

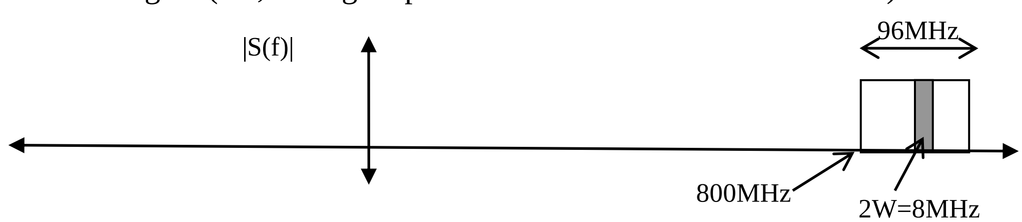
2. For the QCM signal with magnitude response as below, find the least possible band-pass sampling rate f_S. Make a rough plot of the frequency response of the sampled sequence around 0Hz. *Hint*: Use both the band-edges (i.e., f_c+W and f_c-W in order decide the "lowest" sampling rate).



3. In the above problem, let us discuss two possible sources of error:
(a) Assume that the received signal has a phase offset of θ radians; in other words, $s(t) = m_1(t)Cos(2\pi f_c t + \theta) + m_2(t)Sin(2\pi f_c t + \theta)$. Now, what will be the time-domain representation of the sampled sequence? For the special case when $\theta = \pi/2$, what will be the samples of the received signal?
(b) Instead, assume $\theta = 0$ radians, but the incoming carrier frequency f_c' is offset by Δf Hz wrt to the sampling rate designed assuming f_c; i.e., $f_c' = f_c + \Delta f$ and use this f_c' this in place of f_c in the expression for $s(t)$ above. What is then the expression for the time-domain samples after the band-pass sampling ADC in terms of your sampling rate f_S and the Δf ? Say, for Δf = 1MHz, will some of I-Q samples at least be undistorted? Justify your answer.

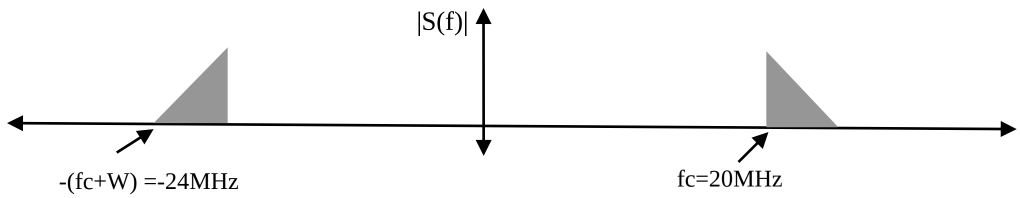
4. A QCM signal $s(t) = m_1(t)Cos(2\pi f_c t) + m_2(t)Sin(2\pi f_c t)$ has the two message signals $m_1(t)$ and $m_2(t)$ of one-sided bandwidth of W_1=3KHz and W_2=4KHz, respectively, and take f_c=31KHz.
(a) Find the minimum band-pass sampling rate $f_s=1/T_s$ that gives un-aliased samples of the two signals.
(b) Assuming that the spectrum of $m_1(t)$ has a "triangular" shape between -3KHz to +3KHz, make a labeled, rough sketch of the spectrum of the samples $m_1(kT_s)$ between -40KHz and +40KHz.

5. A dozen DSB-SC signals of one-sided (low-pass) bandwidth W = 4MHz (including a "guard-band of 0.5MHz) are present between 800MHz and 896MHz, as shown below. Describe the operations (sampling, rate-conversion, filtering) that you need to do to recover Nyquist rate samples of the 7^{th} DSB-SC signal (i.e., the signal present between 848Mz and 856MHz).



6. In the question above, if the dozen signals were QCM, how does your answer change?

7. A real low-pass signal s(t) of one-sided bandwidth W=4MHz is sent as an upper-sideband only single-side band (SSB-SC) signal with magnitude spectrum as shown below. The receiver uses bandpass sampling.



(a) Determine the least sampling rate $f_S = 1/T_S$ required in MHz.

(b) Make a neat plot of the magnitude spectrum |S'(f)| of the sampled sequence $s(kT_S)$ between 24MHz and -24MHz.

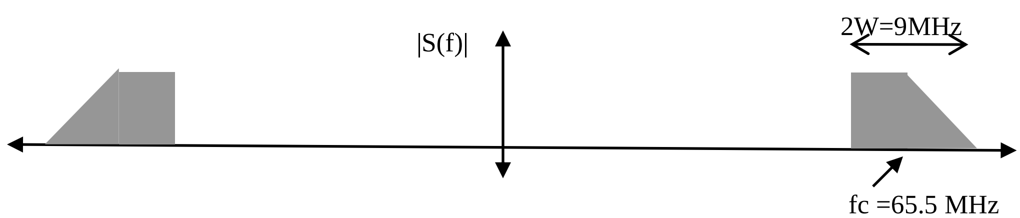
8. A real bandpass QCM signal $s(t) = m_1(t)Cos(2\pi f_c t) + m_2(t)Sin(2\pi f_c t)$ with carrier frequency f_c=200KHz has the two message signals $m_1(t)$ and $m_2(t)$ of one-sided bandwidth of W=5KHz each. Let $S(f)$, $M_1(f)$, and $M_2(f)$ denote the (complex) Fourier transforms of $s(t)$, $m_1(t)$ and $m_2(t)$, respectively. Given $M_1(f\text{=2KHz}) = a + jb$ and $M_2(f\text{=2KHz}) = c + jd$ where j = sqrt(-1), determine the following in terms of a, b, c, and d. The number inside the brackets can be assumed to be in KHz.

(a) $M_1(-2)$ and $M_2(-2)$

(b) $S(202)$ and $S(198)$

(c) $S(-198)$

9. Consider a different QCM signal with magnitude response as below. *Note*: From the information given below, note that both the low-pass message signals have one-sided bandwidth of 4.5MHz each.



(a) Find the least possible band-pass sampling rate f_S for this choice of f_C.

(b) If you were allowed to modify (or design) the carrier frequency between 65MHz and 70MHz given these message signals, what would you choose as f_C to minimize the band-pass sampling rate? Justify your answer.