Department of Electrical Engineering
**Indian Institute of Technology, Madras**

# EE 4140: Digital Communication Systems

September 08 2023 **Tutorial #2** KG/IITM

**1.** Consider the "square" 4-QAM (set x), 16-QAM (set y), and 64-QAM (set z) signal sets.

(a) Plot the 16-QAM signal constellation with "Gray Coding" to ensure that all the nearest neighbor symbols differ only by 1-bit labels.

(b) For the *same* average energy per bit, $E_b$, find the minimum distances $d_x$ of 4-QAM and $d_y$ of 16-QAM in terms of the minimum distance $d_z$ of the 64-QAM constellation.

(c) What is the accurate expression for the average symbol error probability in all the 3 cases?

(d) If $E_b/No = 10$ for all the 3 signals, using the Chernoff bound on the Q-function, calculate the numerical value of the probability of symbol error for the 3 signals.

(e) Reconciling your answers in (a) and (d), what will be the (numerical) value of the bit error probability?

**2.** Consider the signal constellation in Fig. 1 with minimum distance $2d$ is used When this signal is sent through an ideal channel and corrupted by additive white Gaussian noise with variance No/2, and after matched filtering and sampling, the received samples are given by $r(k) = s_i(k) + n(k)$ where $i \in \{1,2,...,6\}$ .

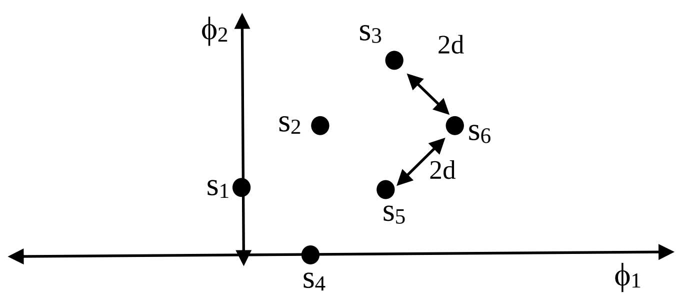



Fig. 1

(a) Assuming that all the symbols are equi-probable, find the exact expression for the average probability of symbol error $P_e$ in the above AWGN channel. Express your answer in terms of $q(d) = Q\left(\frac{d}{\sqrt{N_0/2}}\right)$ with $2d$ as the minimum distance.

(b) Now instead, let the probability of occurrence of symbols $s_1$ and $s_5$ is 1/3, while that of the remaining 4 symbols is 1/12 each. Make a rough plot of the new decision regions, if any.

(c) Can you now find the exact expression for $P_e$ now? Else, use the Union-Bound argument to get an (approximate) expression for $P_e$. Explain the assumptions made.

**3.** Consider the two 8-ary signal constellations in Fig. 2, each with minimum distance $2d$ (which could be different) but with the same average symbol (bit) energy. When either of this signal is sent through an ideal channel and corrupted by additive white Gaussian noise with variance $N_0/2$, and after matched filtering and sampling, the received samples are given by $r(k) = s_i(k) + n(k)$ where $i \in \{1,2,\ldots,8\}$ .

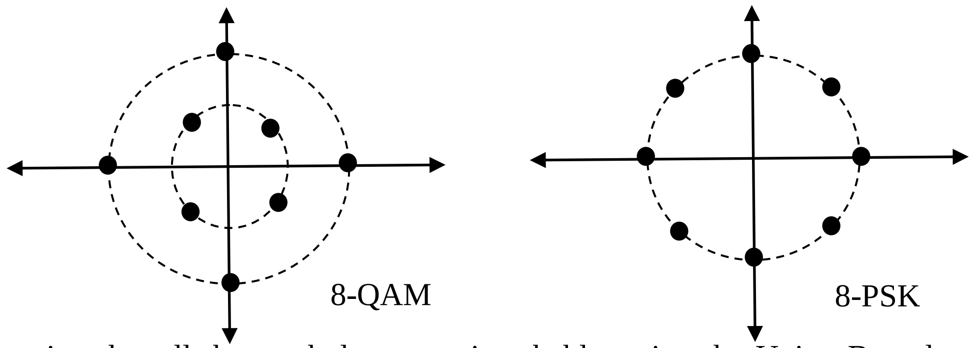


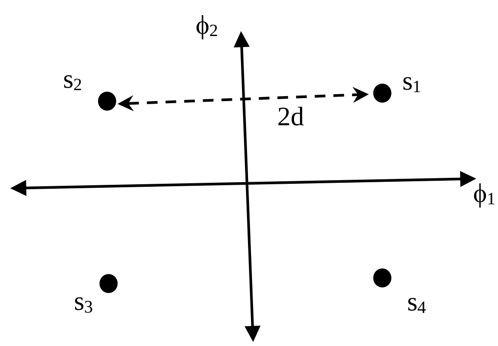

Fig. 2

(a) Assuming that all the symbols are equi-probable, using the Union Bound <u>only over the nearest neighbor(s)</u>, find the expression for the approximate average probability of symbol error $P_e$ in the above AWGN channel for the 8-PSK signal set. Express your answer in terms of $q(d) = Q\left(\frac{d}{\sqrt{N_0/2}}\right)$ with $2d$ as the minimum distance.

(b) Repeat (a) for the circular 8-QAM signal set.

For the same energy per bit, which of the 2 signal sets will have the lower approx. $P_e$ ? Why? Explain.

**4.** Consider the 4-QAM (QPSK) signal constellation below with minimum distance $2d$. When this signal is sent through an ideal channel and corrupted by additive white Gaussian noise with variance No/2, and after matched filtering and sampling, the received samples are given by $r(k) = s_i(k) + n(k)$ where $i \in \{1,2,\ldots,4\}$ .




Given that $N_O/2 = d/8$, and that the prior $P(s_1) = 5/8$, while the other 3 symbols have priors of 1/8 each, make a rough, labelled plot of the <u>optimal decision boundaries</u> required at the receiver. Clearly specify the numerical values of the displacement(s), D , wherever required.

**5.** Consider a band-pass signal $s_{i,j}(t)=d_i(k)\varphi_j(t)$, for $kT \leq t \leq (k+1)T$, where $\varphi_j(t)$, j=1,2,..,$N$, are orthonormal basis functions defined between $0 \leq t \leq T$, and $d_i(k)$ is uniformly drawn from $\{+3d, +1d, -1d, -3d\}$. Given that the total number of signals in the constellation is therefore given by $i \times j = 4 \times N$, answer the following:

(a) For *N*=2, plot the signal constellation.

(b) For *N*=4, what is the exact expression for average probability of symbol error $P_e$ in AWGN channels in terms of *d* and noise variance No/2?

**6.** A good portion of Chap.7 in the text-book ("*Communications Systems Engineering*," Proakis and Salehi) is similar to the material discussed in class. Do the following problems starting with page. 457 in the E-version on the URL.

(a) Problems from **7.10 to 7.15** ,

(b) Problems from **7.21 to 7.33** , and

(c) Problems from **7.42 to 7.53**.