EE4140 **Class # 13** - October 07, 2020

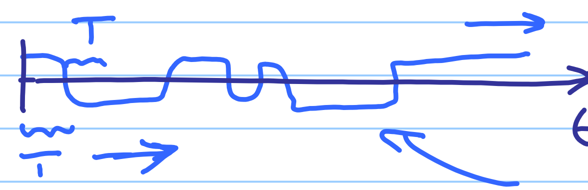
Note Title 3/27/2020

Recall → "Bandwidth" of a digitally modulated signal

$\quad\downarrow$

$PSD \rightleftharpoons ACF$

→ Power / Energy



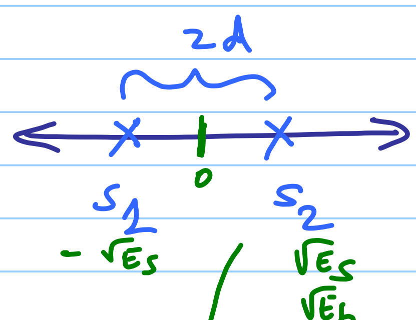
$\underbrace{s_1(t) \cdots s_M(t)}$ — T

$E_s \rightarrow$ Energy per symbol (over T secs)

$E_b \rightarrow$ Energy per bit

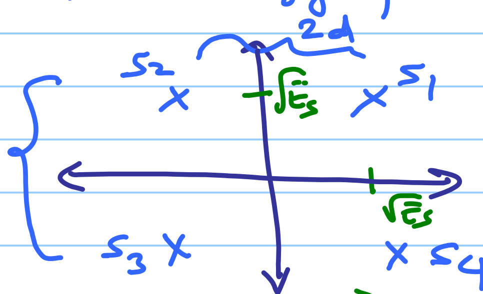
$2d = 2\sqrt{E_s}$

$d = \sqrt{E_s}$



BPSK (2-ary PAM)

$2d = 2\sqrt{E_s} = 2\sqrt{E_b}$



QPSK $= \sqrt{2}\sqrt{E_b}$

(4-ary QAM)

$E_s = 2E_b$

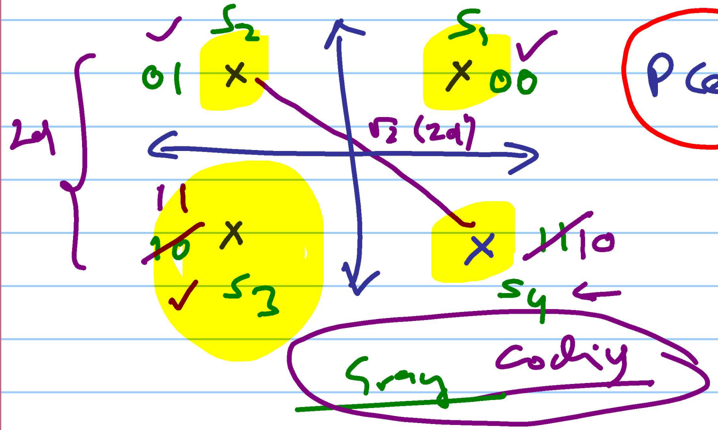
$P(e) = q$

$q = \frac{1}{2}\,\mathrm{erfc}\left(\frac{d}{\sqrt{N_0}}\right)$

$= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$

$P(e) = 2q - q^2$

$q = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{2}\cdot\sqrt{\frac{E_b}{N_0}}\right)$



$P(e) \rightarrow$ Avg. Probability of symbol error $\|$

Bit-error Rate

Symbol-error Rate

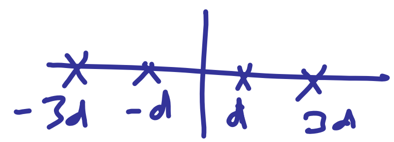
\# of symbol error $\approx$ \# of bit error

$\frac{\cancel{60}\ 30}{2000} \rightarrow \cancel{0.03}\ 0.015$ BER

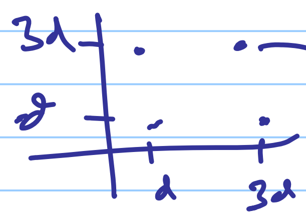
$\frac{30}{1000} = SER$

0.03

$E_b \leftarrow E_s = \sum_{i=1}^{M} p_i a_i^2$



$$\frac{(-3d)^2+(-d)^2+(d^2)+(3d)^2}{4}$$



$(3d+j3d) \to 9d^2+9d^2$

$d_{min} \to d, E_s$

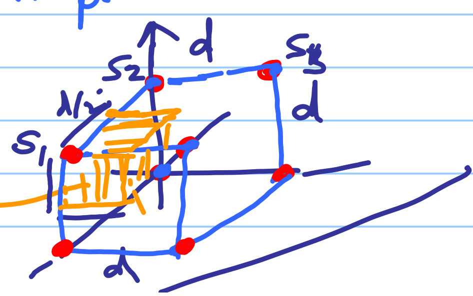
$$\boxed{E_s = \frac{20d^2}{4}}$$

$$\underset{3\times1}{\bar r} = \underset{3\times1}{\bar s_i} + \underset{3\times1}{\bar n}$$

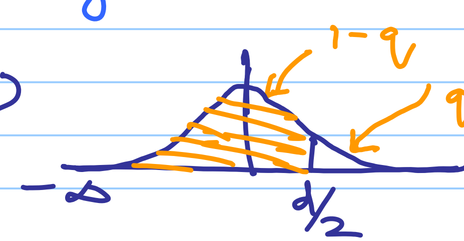
Finding $P(c)$

(*) Example → "Vertices of a Hypercube" → signal waveform generated from a binary code

$r \in Z_1$



$$P(\bar r \in Z_1 | s_1 \text{ sent}) = P_1(c)$$



$$= P\left[-\infty < n_1 < d/2,\ -\infty < n_2 < d/2,\ -\infty < n_3 < d/2\right]$$

$$= \underbrace{\left(P\left[-\infty < n < d/2\right]\right)}^{3}$$

$P_2(c) = (1-q)^3$ $\Rightarrow$ $P(e) = 1 - P(c)$

By Symmetry

$= 1 - (1-q)^3$

$N = 3$

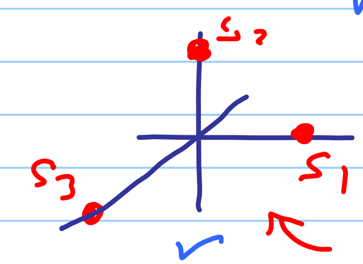
$M = 8$

$\boxed{M = 2^N}$

$\boxed{N = \log_2 M}$

In general, for N-bit binary code $P(e) = \boxed{1 - (1-q)^N}$

* Example : orthogonal signals (FSK)



$\boxed{N = M}$

$\{s_1(t), s_2(t) \cdots s_M(t)\} \rightleftharpoons \{\phi_1(t), \phi_2(t) \cdots \phi_M(t)\}$

$M = N$