Department of Electrical Engineering
Indian Institute of Technology, Madras

EE 4140: Digital Communication Systems

November 06, 2022 **Tutorial #4** KG/IITM

1. A digital communication system operates in a band-limited channel with pass-band of 2W=10MHz. SRRC pulses with 40% excess band-width are used along with a 16-QAM alphabet.

(a) Find the bit-rate of the system.

(b) If the energy per bit is E_b=2 Joules, express the minimum distance d_{min} in terms of E_b.

2. An uniform i.i.d sequence $D(k) \in \{0,1,2,3\}$ is appropriately pre-coded to $P(k)$ and transmitted as 4-ary PAM symbols $I(k)$ after being pulse shaped by a duo-binary filter $g(t)$. Recall that for the duo-binary pulse-shape, $g(kT)$= 1 for k = 0 and 1, and is zero for other values of k, where T is the symbol duration. The discrete-time received sample at time $t=kT$, after appropriate matched-filtering is given by

$$r(k) = I(k) + I(k-1) + n(k)$$

where $n(k)$ is AWGN.

(a) Design a precoder for the channel. Specify the precoder operations. *Hint*: Use base-4 arithmetic.

(b) Make a neat sketch of the decoder decision regions for the noisy measurements $r(k)$.

Hint: To check that your logic is correct, supply it a random input symbol stream $D(k)$ and see if the output decisions match at symbol level. Recollect also that $-m \bmod(M) = (M-m)$, for $m=0,1,2,\ldots(M-1)$.

(c) Indicate the Gray coding on the 4-ary PAM symbols taking the decision regions into account.

3. The received signal thro' a 2-tap channel is given by

$$z(n) = \sum_{l=0}^{1} f_l \, I(n-l) + v(n),$$

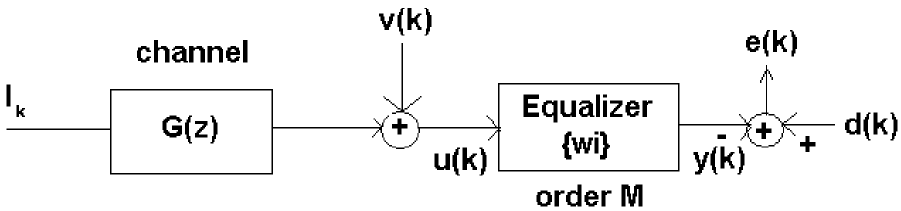
where the FIR channel coefficients f_0=1.0, and f_1=0.6, and data $I(n)$ and noise $v(n)$ are mutually uncorrelated with $I(n) \in \{+1,-1\}$ and the noise is AWGN with variance σ_v^2. The Viterbi algorithm is used to implement MLSE for this measurement model.

(a) Draw a single-stage of the VA, clearly labeling the nodes, and the branches.

(b) The first six values of $z(n)$ are given as follows: $z(1) = 0.5$; $z(2) = 1.1$; $z(3) = 0.1$; $z(4) = -1.2$; $z(5) = 0.3$. Assuming that $I(n) = -1,\ n \le 0$, compute the evolution of the VA over the 5 time-intervals. Indicate the values of the Cumulative Metrics (of all the nodes) at the end of time n=5.

(c) What is the ML sequence as indicated by the VA at the end of time n=5?

4. In the figure below, the input $\{I_k\}$ is i.i.d with $E[I_k^2]=1$ and $P[I_k=+1]=1/2= P[I_k=-1]$. The AWGN v(k) has variance $\sigma_v^2=0.2$ and $E[I_k\ v(i)]=0$ for all k,i.



The channel $G(z)=1-z^{-1}+0.5\ z^{-2}$ and the equalizer has an order equal to M. For the following situations, compute manually **R**, **p**, and eventually $\mathbf{w}_{opt}=\mathbf{R}^{-1}\mathbf{p}$.

a) M=2, d(k)=I(k)
b) M=2, d(k)=I(k-1)
c) M=2, d(k)=I(k-2)
d) What is the J_{min} in each of the above cases?
e) Now, consider a Decision Feedback Equaliser (DFE) with M=2 taps in the feedforward section and 2 taps in the feedback section. For each of the cases in (a), (b), and (c), what will be the resultant **R** and **p** in this case?

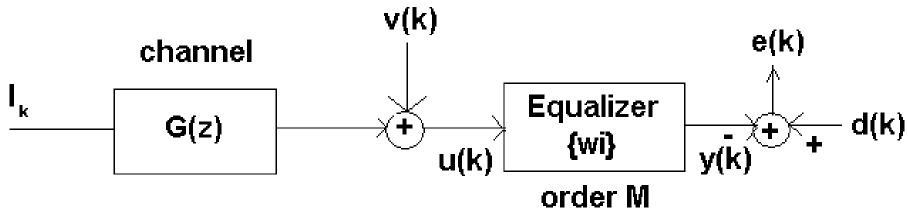
5. An uniform iid sequence $I(n) \in \{-1,+1\}$ is transmitted through a FIR channel $H(z) = 1 - 2z^{-1} + 0.5z^{-2}$ and the resultant output *x(n)* is corrupted by an AWGN sequence *u(n)* with variance $\sigma_u^2 = 0.3$. It is required to define a 2-tap linear equalizer to filter the measurement samples *z(n) = x(n)+u(n).* Assume that {*I(n)*} and {*u(n)*} are mutually uncorrelated.

(a) Specify the auto-correlation matrix clearly.
(b) If the desired sequence is defined by *d(n)= I(n-Δ)*, find the 2-tap linear MMSE equalizer with the least value for Jmin by varying Δ . What is this "best" value for Δ ?
(c) What is the corresponding Jmin for this LE-MMSE?
(d) What is the variance of the residual ISI contribution for this LE-MMSE? If this excess ISI is considered as an additional, uncorrelated, Gaussian noise component, what will be the expression for the average probability of symbol error at the output of the equalizer? (Write this in terms of the $q(d)$ where $q(d) = Q\left(\frac{d}{\sqrt{N_0/2}}\right)$ with $2d$ as the distance between the symbols.)

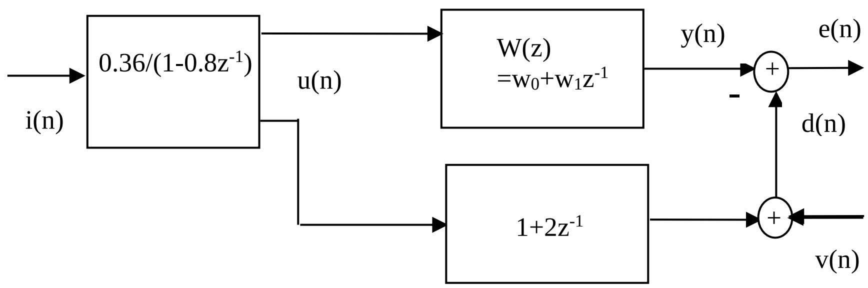
6. A uniform real i.i.d sequence {d[k]} with $E\{|d[k]|^2\}=1$ is filtered by $H(z)= 1-0.5z^{-1}+(1/3)\ z^{-3}$ and the resulting output is corrupted by a colored noise which is a result of AWGN filtered by $1+0.8z^{-1}$ to give the measurements {u[k]} where the variance of the AWGN is 0.25 and this AWGN is uncorrelated with {d[k]}.

a) Find $\mathbf{R}_{uu}$ of size 3×3.
b) Find a 3×1 $\mathbf{p}=E\{\mathbf{u}[k]d[k-\Delta]\}$ for (i)Δ=1, (ii)Δ=4.

7. In the figure below, the input $\{I_k\}$ is i.i.d with $E[I_k^2]=1$ and $P[I_k=+1]=1/2=P[I_k=-1]$. The AWGN v(k) has variance $\sigma_v^2=1$ and $E[I_k\, v(i)]=0$ for all k,I, and the channel response $G(z)=1/(1-0.8z^{-1})$. If **w** is of order M=2, find $\mathbf{w_o}$ for (a) $\sigma_v^2=1$, (b) $\sigma_v^2=0.3$ and (c) $\sigma_v^2=0$.



8. Consider the measurement model below, where a linear MSE (Wiener) estimator is to be defined for the equalizer W(z) which will minimize $E[e^2(n)]$. Here, input i(n) is white noise with unit variance, and additive noise v(n) has variance $\sigma_v^2 = 0.10$ and is uncorrelated with i(n).



(a) Find the correlation matrix **R** and the cross-correlation vector **p**.
(b) What is the LMSE estimate for W(z)?
(c) What is the Jmin for this LMSE problem?

9. For the measurement model below, a decision feedback equalizer w with 3 feed-forward taps and 2 feedback taps is to be defined so as to minimize $E[e^2(k)]$. Here, input symbols $i(n)\epsilon\{+1,-1\}$ are equi-probable, and the additive noise v(k) has variance $\sigma_v^2 = 0.10$ and is uncorrelated with i(n). If the channel $F(z)=1+2z^{-1}-z^{-3}$, and the decoding delay $\Delta=1$, specify the entries of: (a) the auto-correlation matrix **R**, and (b) the cross-correlation vector **p**.