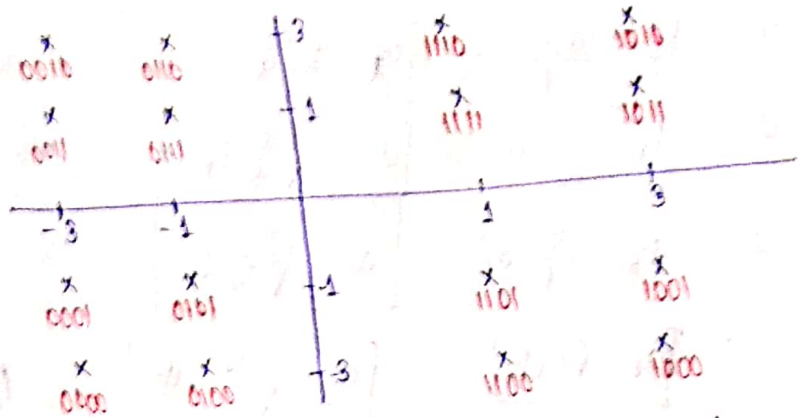
Tutorial 3

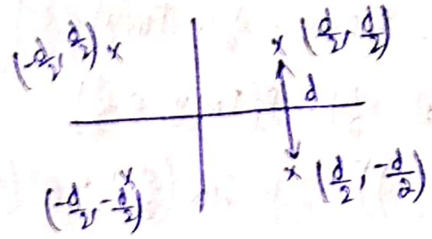
Q.1) (a) 16-QAM constellation with Gray coding:



(b) Given, same E_b. Min. distance of 4-QAM = d_x
Min. distance of 16-QAM = d_y
Min. distance of 64-QAM = d_z

To find d_x and d_y in terms of d_z.

(i) **4-QAM:**

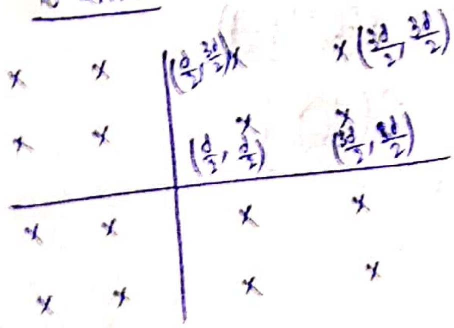


Here, $d = d_x$

$$\text{Avg. symbol energy} = \frac{1}{4}\left[4 \times \left(\frac{d_x^2}{4} + \frac{d_x^2}{4}\right)\right] = \frac{d_x^2}{2}$$

$$E_{b_x} = \frac{1}{2} \times \frac{d_x^2}{2} = \boxed{\frac{d_x^2}{4}} \quad \text{—(1)}$$

(ii) **16-QAM:**



Average symbol energy

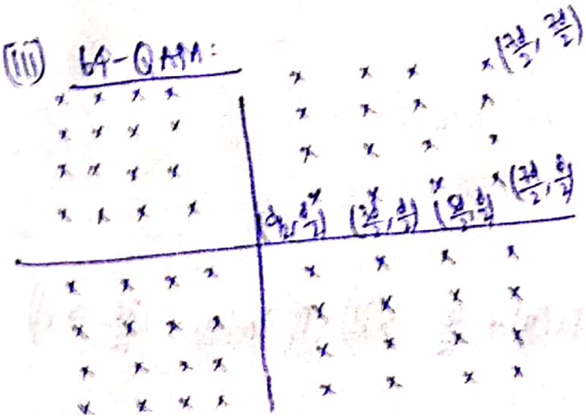
$$= \frac{1}{16}\left[4 \times \left[\frac{2d_y^2}{4} + \frac{10d_y^2}{4} + \frac{10d_y^2}{4} + \frac{18d_y^2}{4}\right]\right]$$

$$= \frac{1}{16}\left[\cancel{4} \times \left[\frac{40d_y^2}{\cancel{4}}\right]\right] = \frac{40d_y^2}{16}$$

$$= \frac{5}{2}d_y^2$$

$$\therefore E_{b_y} = \frac{1}{4} \times \frac{5}{2}d_y^2 = \boxed{\frac{5}{8}d_y^2} \quad \text{—(2)}$$

(iii) **64-QAM:**



Average symbol energy

$$= \frac{1}{64}\left[4 \times \left[\frac{672d_z^2}{4}\right]\right]$$

$$= \frac{672}{64}d_z^2 = \frac{21}{2}d_z^2$$

$$\therefore E_{b_z} = \frac{1}{6} \times \frac{21d_z^2}{2} = \boxed{\frac{7}{4}d_z^2} \quad \text{—(3)}$$

Scanned with CamScanner

It is given that E_b's are same.

i.e, $\boxed{E_{bx} = E_{by} = E_{bz} = E_b}$

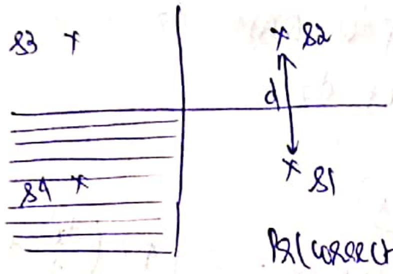
from ① $\Rightarrow$ $E_b = \frac{d_x^2}{4} \Rightarrow \boxed{d_x = 2\sqrt{E_b}}$

from ② $\Rightarrow$ $E_b = \frac{5}{8} d_y^2 \Rightarrow \boxed{d_y = \sqrt{\frac{8}{5} E_b}}$

from ③ $\Rightarrow$ $E_b = \frac{7}{4} d_z^2 \Rightarrow \boxed{d_z = \sqrt{\frac{4}{7} E_b}}$

$\therefore$ $\boxed{d_x = \sqrt{7}\, d_z}$; $\boxed{d_y = \sqrt{\frac{14}{5}}\, d_z}$ Ans:

(c) 4-QAM



The real and imaginary parts are independent and variance σ^2.

$y \to$ received symbol.

$$P(\text{correctly detected} \mid s_4) = P\left(Re(y) < \frac{d}{2},\ Im(y) < \frac{d}{2}\right)$$
$$= P\left(Re(y) < \frac{d}{2}\right) \cdot P\left(Im(y) < \frac{d}{2}\right)$$
$$= \left(1 - Q\left(\frac{d}{2\sigma}\right)\right)^2 = 1 - 2Q\left(\frac{d}{2\sigma}\right) + Q^2\left(\frac{d}{2\sigma}\right)$$

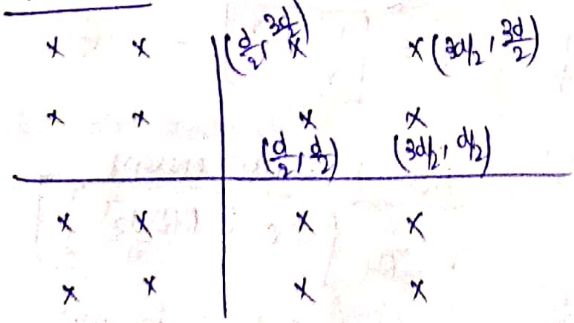
$\therefore$ $P_{e\,(4\text{-QAM})} = 1 - \frac{4\,P(\text{correctly detected} \mid s_4)}{4}$

$$= 1 - 1 + 2Q\left(\frac{d}{2\sigma}\right) + \left(-Q^2\left(\frac{d}{2\sigma}\right)\right)$$

or, $\boxed{P_e(4\text{-QAM}) = 2Q\left(\frac{d}{2\sigma}\right) - Q^2\left(\frac{d}{2\sigma}\right)}$

or, $\boxed{P_e(4\text{-QAM}) \simeq 2Q\left(\frac{d}{2\sigma}\right)}$

16-QAM



There are three different points in the constellation.

(i) Four outermost corner points

$$P(\text{correctly detected} \mid s_x) = P\left(Re(y) + \frac{3d}{2} > d\right) \cdot P\left(Im(y) + \frac{3d}{2} > d\right)$$

$$= P\left(Re(y) > -\frac{d}{2}\right) \cdot P\left(Im(y) > -\frac{d}{2}\right)$$

$$= \left(1 - Q\left(\frac{d}{2\sigma}\right)\right)^2$$

(ii) Eight points (not innermost, not outermost)

$$P(c|s_1) = P\left(d < Re(y) + \frac{3d}{2}\right) \cdot P\left(0 < Im(y) + \frac{d}{2} < d\right)$$

$$= P\left(Re(y) > -\frac{d}{2}\right) \cdot P\left(-\frac{d}{2} < Im(y) < \frac{d}{2}\right)$$

$$= \left[1 - Q\left(\frac{d}{2\sigma}\right)\right] \cdot \left[1 - 2Q\left(\frac{d}{2\sigma}\right)\right]$$

(iii) Innermost four points

$$P(c|s_1) = P\left(0 < Re(y) + \frac{d}{2} < d\right) \cdot P\left(0 < Im(y) + \frac{d}{2} < d\right)$$

$$= P\left(-\frac{d}{2} < Re(y) < \frac{d}{2}\right) \cdot P\left(-\frac{d}{2} < Im(y) < \frac{d}{2}\right)$$

$$= \left(1 - 2Q\left(\frac{d}{2\sigma}\right)\right)^2.$$

$$\therefore P(c) = \frac{1}{16}\left(4\left[1 + Q^2\left(\frac{d}{2\sigma}\right) - 2Q\left(\frac{d}{2\sigma}\right)\right] + 8\left[1 - 2Q\left(\frac{d}{2\sigma}\right) - Q\left(\frac{d}{2\sigma}\right) + 2Q^2\left(\frac{d}{2\sigma}\right)\right] + 4\left[1 + 4Q^2\left(\frac{d}{2\sigma}\right) - 4Q\left(\frac{d}{2\sigma}\right)\right]\right)$$

$$\therefore P(c) = \frac{1}{16}\left[4 + 4Q^2\left(\frac{d}{2\sigma}\right) - 8Q\left(\frac{d}{2\sigma}\right) + 8 - 16Q\left(\frac{d}{2\sigma}\right) - 8Q\left(\frac{d}{2\sigma}\right) + 16Q^2\left(\frac{d}{2\sigma}\right) + 4 + 16Q^2\left(\frac{d}{2\sigma}\right) - 16Q\left(\frac{d}{2\sigma}\right)\right]$$

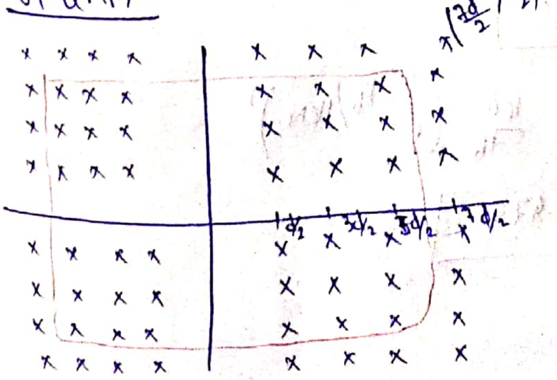
$$= \frac{1}{16}\left[16 + 36Q^2\left(\frac{d}{2\sigma}\right) - 48Q\left(\frac{d}{2\sigma}\right)\right]$$

$$= 1 + \frac{9}{4}Q^2\left(\frac{d}{2\sigma}\right) - 3Q\left(\frac{d}{2\sigma}\right)$$

$$\therefore \boxed{P_e = 1 - P(c) = 3Q\left(\frac{d}{2\sigma}\right) - \frac{9}{4}Q^2\left(\frac{d}{2\sigma}\right)}$$

or, $\boxed{P_{e\,(16\text{-}QAM)} \simeq 3Q\left(\frac{d}{2\sigma}\right)}$

64-QAM:



There are 3 types of points:

(i) Outermost corner 4 points:

$$P(c|s_1) = P\left(Re(y) > -\frac{d}{2}\right)^2 = \left(1 - Q\left(\frac{d}{2\sigma}\right)\right)^2$$

(ii) Inner 36 (marked):

$$P(c|s_1) = \left(P\left(-\frac{d}{2} < Re(y) < \frac{d}{2}\right)\right)^2$$

$$= \left(1 - 2Q\left(\frac{d}{2\sigma}\right)\right)^2$$

(iii) Outermost pts except corner pts (24):

$$P(C|s_k) = P\left(-\frac{d}{2} < Re(y) < \frac{d}{2}\right) \cdot P\left(Im(y) > -\frac{d}{2}\right)$$

$$= \left(1 - Q\left(\frac{d}{2\sigma}\right)\right)\left(1 - 2Q\left(\frac{d}{2\sigma}\right)\right).$$

$$\therefore P(C) = \frac{1}{64}\left[4 + 4Q^2\left(\frac{d}{2\sigma}\right) - 8Q\left(\frac{d}{2\sigma}\right) + 24 - 48Q\left(\frac{d}{2\sigma}\right) - 24Q\left(\frac{d}{2\sigma}\right) + 48Q^2\left(\frac{d}{2\sigma}\right) + 36 + 144Q^2\left(\frac{d}{2\sigma}\right) - 144Q\left(\frac{d}{2\sigma}\right)\right]$$

$$= \frac{1}{64}\left[64 + 196Q^2\left(\frac{d}{2\sigma}\right) - 224Q\left(\frac{d}{2\sigma}\right)\right]$$

$$\therefore P(C) = 1 - \frac{7}{2}Q\left(\frac{d}{2\sigma}\right) + \frac{49}{16}Q^2\left(\frac{d}{2\sigma}\right)$$

$$\therefore \boxed{P_{e\,(64\text{-}QAM)} = 1 - P(C) = \frac{7}{2}Q\left(\frac{d}{2\sigma}\right) - \frac{49}{16}Q^2\left(\frac{d}{2\sigma}\right)}$$

or, $\boxed{P_{e(64\text{-}QAM)} \simeq \frac{7}{2}Q\left(\frac{d}{2\sigma}\right)}$

From (b),

$$P_{e(4\text{-}QAM)} = 2Q\left(\frac{d}{2\sigma}\right) - Q^2\left(\frac{d}{2\sigma}\right) = 2Q\left(\sqrt{\frac{E_b}{N_0}}\right) - Q^2\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_{e(16\text{-}QAM)} = 3Q\left(\frac{d}{2\sigma}\right) - \frac{9}{4}Q^2\left(\frac{d}{2\sigma}\right) = 3Q\left(\sqrt{\frac{2E_b}{5N_0}}\right) - \frac{9}{4}Q^2\left(\sqrt{\frac{2E_b}{5N_0}}\right)$$

$$P_{e(64\text{-}QAM)} = \frac{7}{2}Q\left(\frac{d}{2\sigma}\right) - \frac{49}{16}Q^2\left(\frac{d}{2\sigma}\right) = \frac{7}{2}Q\left(\sqrt{\frac{E_b}{7N_0}}\right) - \frac{49}{16}Q^2\left(\sqrt{\frac{E_b}{7N_0}}\right)$$

Ans

(d). Chernoff-bound: $\boxed{Q(x) \le e^{-x^2/2}}$ $\qquad \frac{E_b}{N_0} = 10$.

$$\rightarrow P_{e(4QAM)} \le 2e^{-E_b/2N_0} - \left(e^{-E_b/2N_0}\right)^2$$

or, $\boxed{P_{e(4\text{-}QAM)} \le 0.013}$

$$\rightarrow P_{e(16\text{-}QAM)} \le 3e^{-\frac{2E_b}{10N_0}} - \frac{9}{4}\left(e^{-\frac{1}{5}\frac{E_b}{N_0}}\right)^2$$

or, $\boxed{P_{e(16\text{-}QAM)} \le 0.365}$

$$\rightarrow P_{e(64\text{-}QAM)} \le \frac{7}{2}e^{-E_b/14N_0} - \frac{49}{16}\left(e^{-E_b/14N_0}\right)^2$$

or, $\boxed{P_{e(64\text{-}QAM)} \le 0.979}$

(i) Numerical value for bit error probability

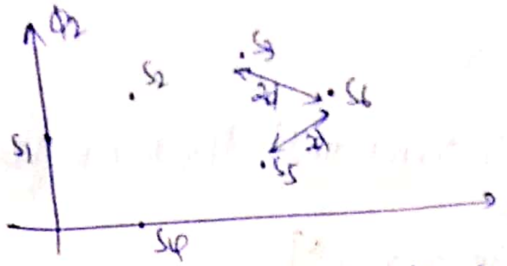
$$P_{\text{bit error}}(4\text{-QAM}) \le \frac{0.013}{2} = 0.0065$$

$$P_{\text{bit error}}(16\text{-QAM}) \le \frac{0.365}{4} = 0.0913$$

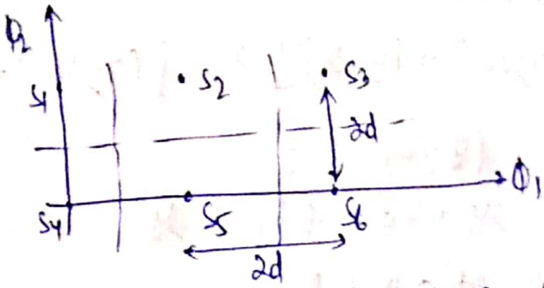
$$P_{\text{bit error}}(64\text{-QAM}) \le \frac{0.979}{6} = 0.1632$$

Ans

3.2) Given,



We know, rotation of a constellation doesn't change the P_e. So, rotating the above constellation, we get:



(a). Considering the constellation points S_1, S_3, S_4 and S_6, we have

$$P_c = (1-q)(1-q)$$

or, $\boxed{P_c = (1-q)^2}$

and, considering the points S_2 and S_5, we have

$\boxed{P_c = (1-q)(1-2q)}$, where $q = q(d) = Q\left(\frac{d}{\sqrt{N_0/2}}\right)$

$$\therefore P_c(\text{avg}) = \frac{1}{6}\left[(1-q)^2 \times 4 + (1-q)(1-2q)\times 2\right]$$

$$= \frac{1}{6}\left[(1+q^2-2q)\times 4 + (1-3q+2q^2)\cdot 2\right]$$

$$= \frac{1}{6}\left[4+4q^2-8q+2-6q+4q^2\right]$$

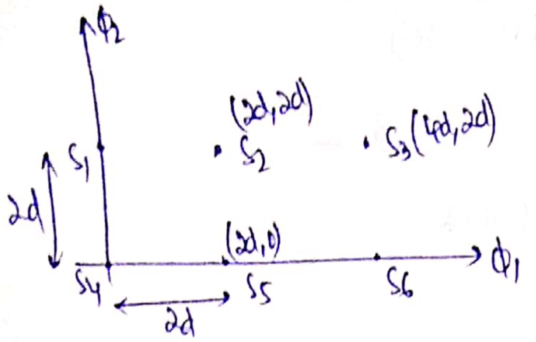
$$= \frac{1}{6}\left[6-14q+8q^2\right]$$

or $P_e(\text{avg}) = 1 - P_c(\text{avg}) = 1 - \left[1 - \frac{14}{6}q + \frac{8}{6}q^2\right]$

$$= \frac{7}{3}q - \frac{4}{3}q^2$$

or, $\boxed{P_e(\text{avg}) = \frac{1}{3}\left[7q - 4q^2\right]}$ Ans.

(b) Now given, $P(S_2) = P(S_5) = \frac{1}{3}$ & $P(S_1) = P(S_4) = P(S_3) = P(S_6) = \frac{1}{12}$



We consider the point S_2 and find its decision boundary with the points S_5 & S_3.

(i) Boundary with S_5:

P('S' received | S_2 transmitted) . P(S_2 transmitted) > P('S' received | S_5 transmitted) · P(S_5 transmitted).

or, $$\cancel{\frac{1}{3}} \cdot \exp\left(-\frac{[(x-2d)^2+(y-2d)^2]}{\cancel{2}\cdot\frac{N_0}{\cancel{2}}}\right) > \cancel{\frac{1}{3}} \exp\left(-\frac{[(x-2d)^2+(y-0)^2]}{\cancel{2}\cdot\frac{N_0}{\cancel{2}}}\right)$$

or, Taking ln both sides,

$$-[\cancel{(x-2d)^2}+(y-2d)^2] > -[\cancel{(x-2d)^2}+y^2]$$

or, $(y-2d)^2 < y^2$

or, $\cancel{y^2}+4d^2-4dy < \cancel{y^2}$

or, $4d(d-y) < 0$

or, $d-y < 0$

or, $\boxed{y>d}$ — boundary with S_5.

(ii) Boundary with S_3:

P('S' received | S_2 transmitted) . P(S_2 transmitted) > P('S' received | S_3 transmitted) · P(S_3 transmitted)

or, $$\frac{1}{\cancel{3}} \cdot \exp\left(-\frac{[(x-2d)^2+(y-2d)^2]}{\cancel{2}\cdot\frac{N_0}{\cancel{2}}}\right) > \frac{1}{\cancel{12}_{4}} \exp\left(-\frac{[(x-4d)^2+(y-2d)^2]}{\cancel{2}\cdot\frac{N_0}{\cancel{2}}}\right)$$

or, $$\exp\left(-\frac{[(x-2d)^2+(y-2d)^2]}{N_0}\right) > \frac{1}{4}\exp\left(-\frac{[(x-4d)^2+(y-2d)^2]}{N_0}\right)$$

Taking ln both sides,

$$-\frac{[(x-2d)^2+\cancel{(y-2d)^2}]}{N_0} > -\ln 4 - \frac{[(x-4d)^2+\cancel{(y-2d)^2}]}{N_0}$$

or, $$\frac{(x-2d)^2}{N_0} < \ln 4 + \frac{(x-4d)^2}{N_0}$$

or, $\cancel{x^2}+4d^2-4xd < N_0\ln 4 + \cancel{x^2}+16d^2-8xd$

or, $4xd - 12d^2 < N_0 \ln 4$

or $4d(x-3d) < N_0 \ln 4$

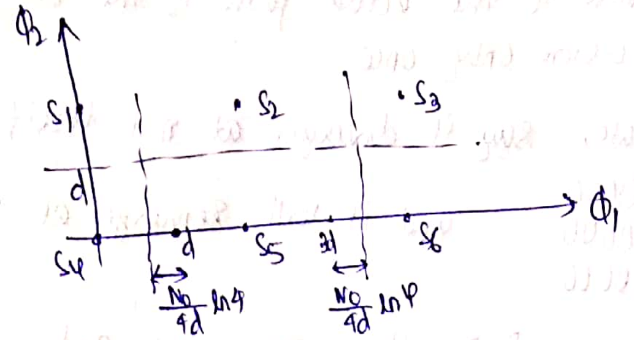
or $x - 3d < \frac{N_0 \ln 4}{4d}$

or, $\boxed{x < \frac{N_0 \ln 4}{4d} + 3d}$ — boundary with S_3.

Similarly if we consider the boundary of S_2 with S_1 we would get:

$\boxed{x > d - \frac{N_0}{4d}\ln 4}$ — boundary with S_1

Plot of the new decision region:



(c). Now,

$$P_c(\text{avg}) = \frac{4}{12}\left[Q\left(\frac{d}{\sigma}\right) Q\left(\frac{-\frac{N_0}{4d}\ln 4}{\sigma}\right)\right] + \frac{2}{3}\left[Q\left(\frac{d}{\sigma}\right)\left(1 - 2Q\left(\frac{\frac{N_0}{4d}\ln 4}{\sigma}\right)\right)\right]$$

or, $P_c = \frac{1}{3}\left[Q\left(\frac{d}{\sigma}\right)\left[1 - Q\left(\frac{\sigma}{2d}\ln 4\right)\right]\right] + \frac{2}{3}\left[Q\left(\frac{d}{\sigma}\right)\left(1 - 2Q\left(\frac{\sigma}{2d}\ln 4\right)\right)\right]$

$\left(\because \sigma^2 = \frac{N_0}{2}\right)$.

or, $P_c = \frac{1}{3}Q\left(\frac{d}{\sigma}\right) - \frac{1}{3}Q\left(\frac{d}{\sigma}\right)Q\left(\frac{\sigma}{2d}\ln 4\right) + \frac{2}{3}Q\left(\frac{d}{\sigma}\right) - \frac{4}{3}Q\left(\frac{d}{\sigma}\right)Q\left(\frac{\sigma}{2d}\ln 4\right)$

or, $P_c = Q\left(\frac{d}{\sigma}\right) - \frac{5}{3}Q\left(\frac{d}{\sigma}\right)Q\left(\frac{\sigma}{2d}\ln 4\right)$

& $P_e = 1 - P_c$

or, $P_e = 1 - Q\left(\frac{d}{\sigma}\right) + \frac{5}{3}Q\left(\frac{d}{\sigma}\right)Q\left(\frac{\sigma}{2d}\ln 4\right)$

or, $\boxed{P_e = 1 - Q\left(\frac{d}{\sigma}\right)\left[1 - \frac{5}{3}Q\left(\frac{\sigma}{2d}\ln 4\right)\right]}$ Ans:

Q.5)

$$z(n) = \sum_{l=0}^{L-1} c_l\, I(n-l) + w(n)$$

$L = 6$; QPSK symbol is sent $\Rightarrow$ $(M=4)$.

(a). States (nodes) in VA:

Total no. of states $= M^{L-1} = 4^5$

(b). Transition metric computations required at every symbol time:

4×4^5 computations $= 4^6$ TM computations

(c). 10 symbol periods, all $1+j1$ sequence was transmitted

Let
00 – $1+j1$ (0)
01 – $-1+j1$ (1)
10 – $-1-j1$ (2)
11 – $1-j1$ (3)

For easy representation we use decimal representation.

The closest path to the actual path is the one that makes a wrong decision only once.

From 00000 state, say it diverges at $n=1$ itself, it may go to
10000
20000
30000
and it will remerge at $n=6$.

$\therefore$ 3 possible sequences we get

$\therefore$ It can diverge at $n = 2, 3, 4, 5$ time instants so that it can re-merge at the end.

$\therefore$ $5 \times 3 =$ 15 possible sequences.

6.

Given that,

$$z(n) = 0.9\, I(n) + 0.4\, I(n-1) + v(n)$$

$$\boldsymbol{z} = [\, z(1),\; z(2), \dots,\; z(6)\,] = [\, -1.1,\;\; 0.4,\;\; 1.5,\;\; 1.2,\;\; -0.6,\;\; -1.2\,]$$

$$N = 6, \qquad L = 2$$

$$I(n) \in \{-1,1\} \;\Rightarrow M = 2$$

$$\Rightarrow \boldsymbol{I}^N \in \{-1,1\}^6$$

$$\#possible\ sequences = M^N = 64$$

(a)

Fig. 6.1. shows the Trellis for a single stage of the Viterbi Algorithm (VA). Note that the number of states at each time instance is $M^{L-1} = 2$. For simplicity, the states and transitions corresponding to the symbol $I(k) = -1$ are marked as 0. Also, the values of $z_{p\to q}(k) = 0.9\, I_q(k) + 0.4\, I_p(k-1)$ are provided adjacent to the branches of the Trellis. These are required to compute the transition metrics $TM_{p\to q}(k)$.

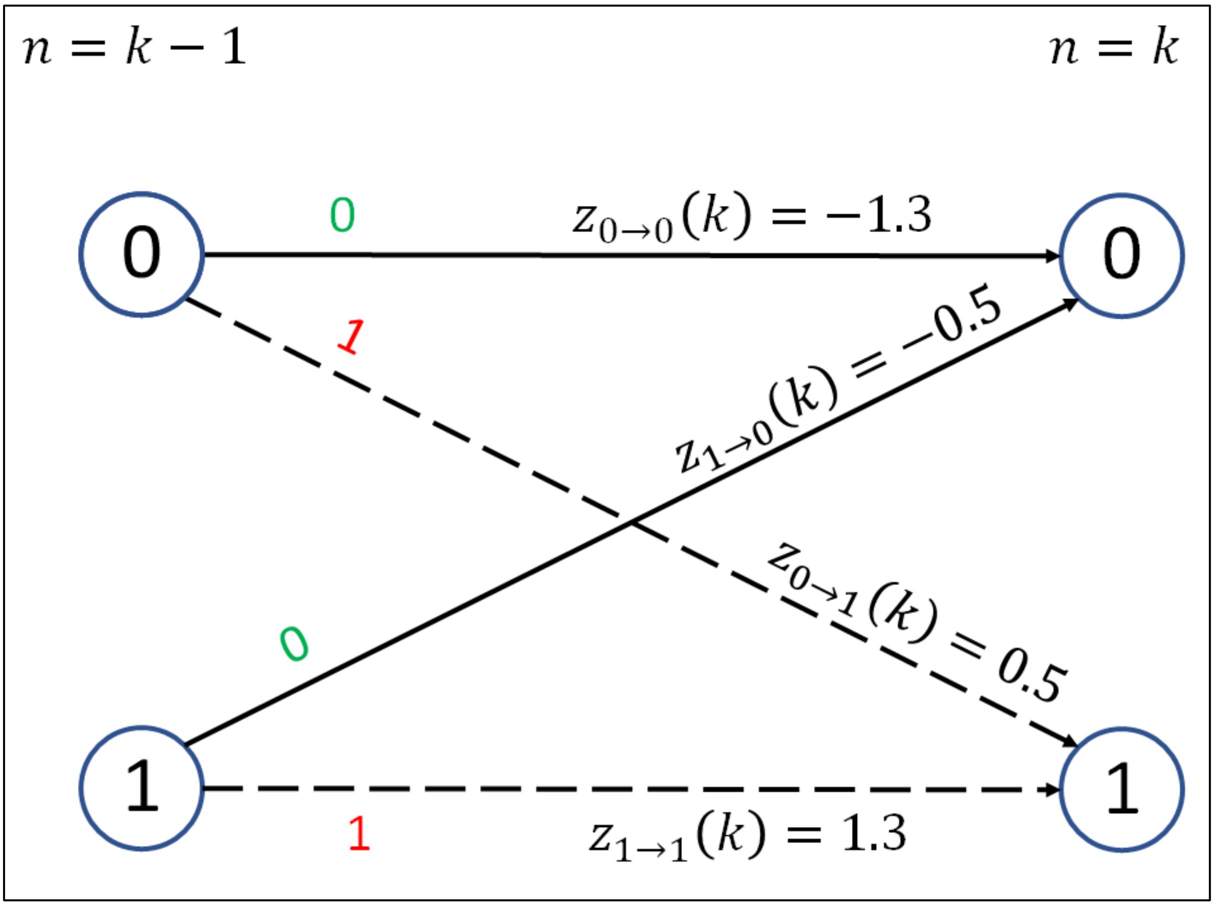


Fig. 6.1. Trellis for a single stage of the VA.

(b)

For the given sequence of received signal samples $\boldsymbol{z}$, the VA is performed with the assumption that $I(0) = -1$ in order to recover the symbol sequence $[\,I(1),\ I(2), \ldots,\ I(6)]$. The stages of the algorithm are shown in Fig. 6.2. Based on the incoming samples $z(k)$ and using $z_{p\to q}(k)$ provided in Fig. 6.1., the transition metrics $TM_{p\to q}(k) = \left(z(k) - z_{p\to q}(k)\right)^2$ are computed (marked in PURPLE adjacent to corresponding branches). Using these transition metrics and the assumption that $CM_0(0) = 0$, cumlative metric corresponding to the node 'q' at time 'k' are computed as $CM_q(k) = \min_{p\in\{0,1\}} CM_p(k-1) + TM_{p\to q}(k)$ and are provided in BLUE adjacent to the nodes. The branches which survived are marked in YELLOW.

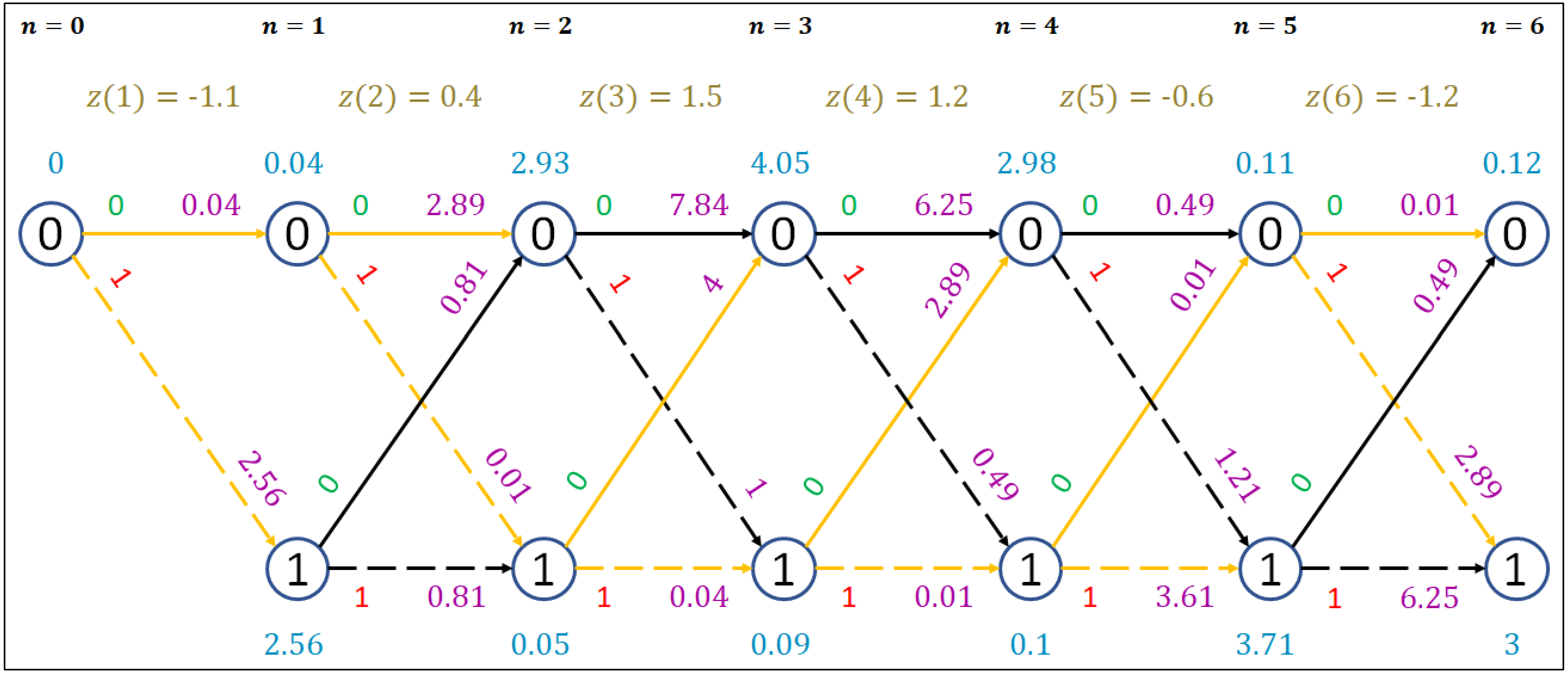


Fig. 6.2. VA for the given sequence of the received signal samples $[\,z(1),\ z(2), \ldots,\ z(6)\,]$.

(c)

By finding the node having the least cumulative cost at stage $n = 6$ and tracing back the survivors which resulted in that cost (as shown in Fig. 6.3.), the Maximum Likelihood sequence is estimated as

$$\boxed{\hat{\boldsymbol{I}}^N = \left[\hat{I}(1),\ \hat{I}(2),\ \ldots,\ \hat{I}(6)\right] = [\,-1,\ 1,\ 1,\ 1, -1, -1\,]}$$

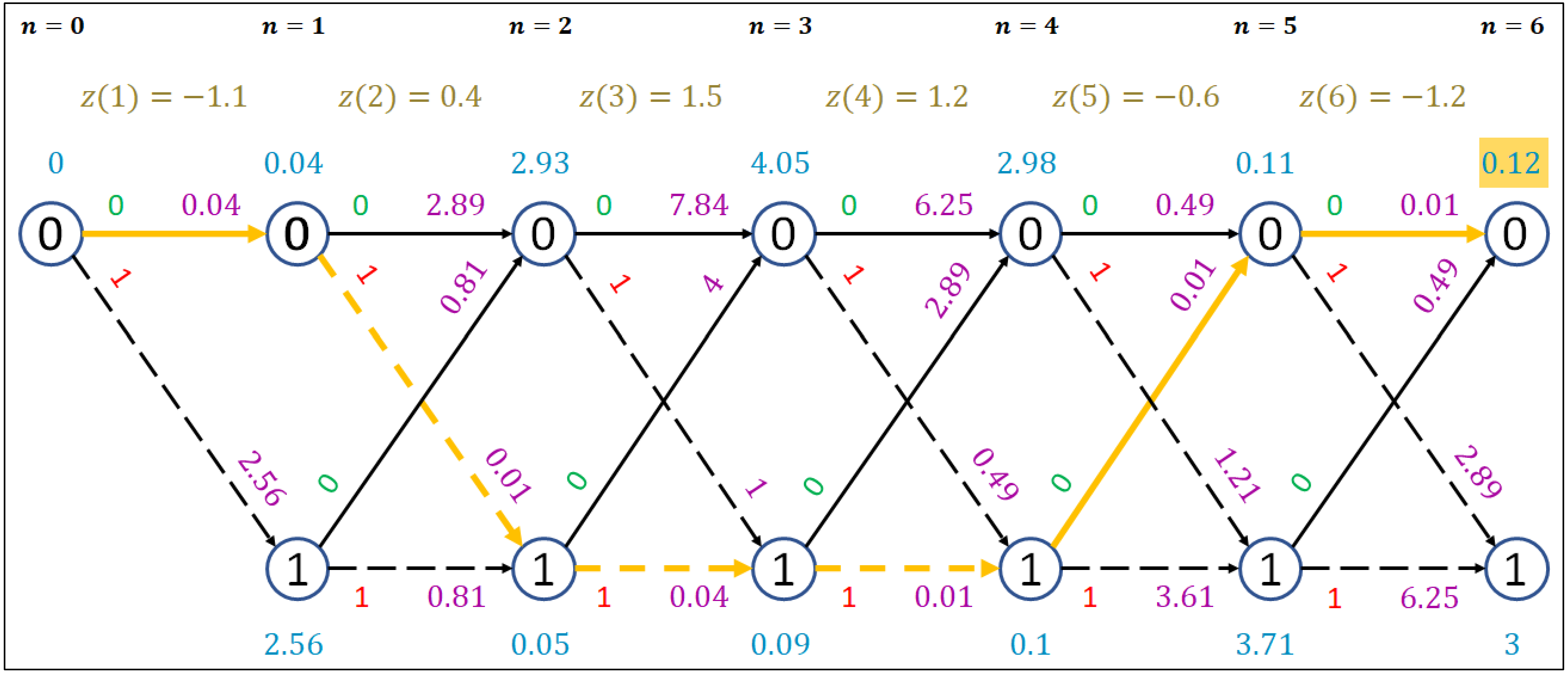


Fig. 6.3. Resulting MLSE sequence from the VA for the given sequence [$z(1)$, $z(2)$, ... , $z(6)$].

To verify this result, we shall search over $M^N = 64$ sequences and find the sequence of symbols which minimizes $\sum_{n=1}^{6}[z(n) - 0.9\, I(n) - 0.4\, I(n-1)]^2$. The simulation is done in MATLAB and the code is shown below.

```
clc; clear all; close all;
M=2;
f=[0.9 0.4];
z=[-1.1 0.4 1.5 1.2 -0.6 -1.2];
L=length(f);
N=length(z);

% list of possible binary sequences
seq_lib=fliplr(de2bi(0:M^N-1,log2(M^N)));
for ii=1:M^N
    z_h(ii,:)=conv(f,[-1 2*seq_lib(ii,:)-1]);
end
z_h(:,[1 end])=[];
[~,mlse_idx]=min(vecnorm((z-z_h)'));
mlse_seq=seq_lib(mlse_idx,:);
fprintf('Sequence with Maximum Likelihood (MLSE):\n');
disp(2*mlse_seq-1);
```

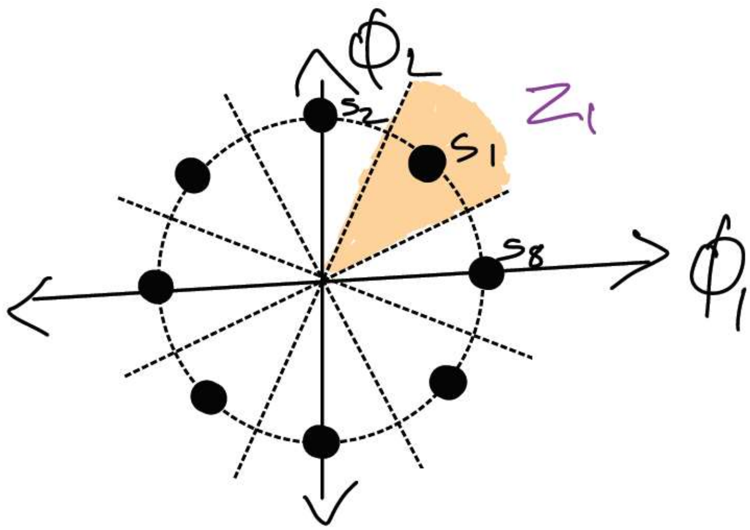
The result obtained is as follows.

```
Sequence with Maximum Likelihood (MLSE):
    -1     1     1     1    -1    -1
```

Thus, the sequence estimated by the VA matches with the one obtined with M^N dimensional search.

3.

(a) 8-PSK



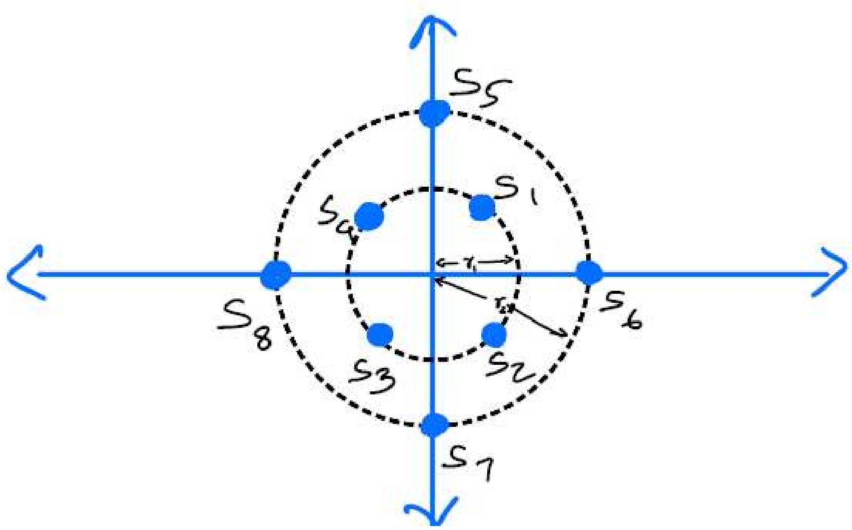
Distance from s1 to nearest neighbors s2 and s8 is 2d. Then, probability of error,

$Pe = Pe_{12}+Pe_{18} = 2q(d)$

This follows from the lecture.

Minimum distance for 8-PSK is 2 sqrt(Es) sin(pi/8)

(b) Circular 8-QAM



We have 8 constellation points. For the circular 8-QAM constellation shown, four of them are along the smaller radius r1 = sqrt(2) and four along the larger radius r2 = 1+sqrt(3). So, we calculate error probability of one symbol from each of the two sets. For example, we can choose s1 from the inner circle and s5 from the outer circle. The coordinates of s1 are (1,1) and that of s5 are (0,1+sqrt(3)).

We consider s1 and try to find its nearest neighbors by calculating the distance to the four symbols near it. The distance to s4 and s2 can be shown to be 2 and the distance to s5 and s6 can also be shown to be 2. Thus, all four of these points are at minimum distance from s1. Then, the probability of error for s1 is $4q(d_{min}) = 4q(2)$.

The outer points have two nearest neighbors, and so their probability of error is $2q(d_{min})$. Thus, for the modulation scheme, *average* probability of symbol error is $3*q(d_{min}) = 3q(2) = 0.0683$.

Average symbol energy for 8-QAM is $E_{s,\,avg}$ **=** $(r_1^2+r_2^2)/2 = 3+\mathrm{sqrt}(3)$

For the same energy per bit (equivalently, energy per symbol as both constellations have the same number of bits), 8-PSK needs the minimum distance: $d_{min}^{8PSK} = 2\ \mathrm{sqrt}(3+\mathrm{sqrt}(3))\ \sin(pi/8)$

Here we substitute the average symbol energy for 8QAM in the expression of minimum distance for 8PSK. The above expression evaluates to 1.6649. Thus, the corresponding probability of symbol error for 8PSK is 2q(1.6649) = 0.0959.

From the probability of error values, it can be concluded that 8QAM has lower approximate Pe. This is justified by the fact that it has a greater minimum distance for the same symbol energy. The minimum distance is enough to offset the fact that 8QAM has a larger number of nearest neighbors.

4. (a)

B/W = 2 MHz, QPSK symbols and sinc pulse shaping (Beta=0)

Bit rate = (Bandwidth/(1+Beta))*log2(M) where M is the number of constellation points.

Here, bit rate = 4 Mbps.

(b) For Beta=0.5, to preserve Bit rate = 4 Mbps,

Log2(M) = 4*(1.5/2)=3 or M=8.

So we need 8PSK.

For Beta =1, Log2(M) = 4*(2/2)=4 or M=16.

We need 16-PSK to preserve the bit rate for Beta=1.