*Department of Electrical Engineering, IIT Madras*
EE-4140: Digital Communication Systems

**Due by Nov. 11, 2022**

**October 2022** **Simulation Assignment #2** **25 marks**

**MLSE & Viterbi Algorithm**: Consider a distorting (ISI) channel defining the measurement model as:

$$r(k) = \sum_{l=0}^{L-1} f_l \; I(k-l) + v(k)$$

Here, the independent, uniformly distributed data symbols $I(k)$ entering the channel $F(z)$ are drawn from a 4-ary PAM alphabet, and the transmit signal power $E[|I(k)|^2] = \sigma_I^2 = 1$. The $L$-tap channel is specified for $L$=3 by the transfer function $F(z) = \frac{1}{\sqrt{2}}(0.8 - z^{-1} + 0.6z^{-2})$ and the additive white, Gaussian, noise component $v(k)$ is zero mean with variance $\sigma_v^2$. The signal to noise ratio (SNR) measured on $r(k)$, is then given by $1/\sigma_v^2$ (why?), and we vary the SNR on the measurements by varying $\sigma_v^2$ appropriately. We study the performance of a MLSE based approach implemented using the Viterbi Algorithm (VA).

**(a) [2+2+2+2+2 = 10 marks]** Define a VA based sequence estimator, assuming perfect information about $F(z)$ is available at the receiver. Generate for each SNR (in 2dB steps from 0dB to 16dB), say 100,002 symbols of 4-PAM using uniform random variables appropriately to get $I(k)$. The noise $v(k)$ is generated using the normal pdf, and the value returned is scaled using the current $\sigma_v$ (for the given SNR). The last 2 symbols (note: $L$-1 = 2 here for $F(z)$) of $I(k)$ may be viewed as "tail symbols" which are known to the receiver, to terminate the VA in a known state. Measure SER only over the (unknown) remaining 100,000 symbols, for the following choices of "trace-back length" or decoding delay δ, namely: (a1) δ=3; (a2) δ=6; (a3) δ=10; (a4) δ=20 and (a5) δ=40; Compute the $\log_{10}$SER versus $10\log_{10}$(SNR) for each choice of δ and plot all results in Fig 1.

**(b) [1+4+4+5 = 10 marks]** Now, consider a different FIR channel $G(z)$ with $L$=6 taps specified by the transfer function

$$G(z) = \frac{1}{C}(1 - 0.95z^{-1} + 0.5z^{-2} + 0.15z^{-3} - 0.2z^{-4} - 0.1z^{-5})$$

where the transmit symbols here are 2-ary PAM with $E[|I(k)|^2] = \sigma_I^2 = 1$. Now, answer the following:

(b1) Normalise the channel gain to unity by appropriate choice of $C$ in $G(z)$. What is this $C$ ?

(b2) As in part (a), vary SNR by changing noise variance appropriately. Over 0dB to 28dB, evaluate the SER of the $2^5$ state VA by sending 100,004 symbols in each step of 4dB. Using decoding delay δ=30, compute the $\log_{10}$SER versus $10\log_{10}$(SNR) and plot the results in Fig 2.

(b3) Now, suppose, the VA is constructed by considering a truncated impulse response, where only the first 3 taps of $G(z)$ are taken into consideration, to give a $2^2$ state VA (i.e., the weaker taps, namely 0.15, -0.2, and -0.1 are ignored in constructing the VA). Note, however, that the measurements would have contribution from all the 6-taps of $G(z)$. For this case, evaluate and plot $\log_{10}$SER versus $10\log_{10}$(SNR), on the Fig 2 as in (b2), for δ=30.

(b4)* Now, can you incorporate a decision-feedback mechanism to improve the performance of this $2^2$ state VA? *Hint*: Sub-optimal low-delay decisions can be made at each state based on the corresponding survivor sequence for that state, and this contribution can be subtracted before computing the transition metrics. If you need more details, read the DDFSE paper by Duel-Hallen and Heegard (1989). Plot this result for δ=30 also in Fig.2, and comment on your results. *indicates higher order to difficulty

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*Instructions*

**Submit only a hand-written report.** Your name and roll-number must appear in the first page. All plots can be printed and attached to your report. Your working code must be properly commented and be emailed to the TAs before the due date. Their email address are: Sruti at ee18d705@smail.iitm.ac.in and Prasikaa at ee21d700@smail.iitm.ac.in. Your working code can be named "rollnumber-assignment2-code.m". The TAs will get back to you if required. Please see other instructions, if any, in the WhatsApp group.