Due before Monday, 10am, Sep. 12, 2022

Department of Electrical Engineering, IIT Madras

EE-4140: Digital Communication Systems

Sep. 04, 2022 **Simulation Assignment #1 (Edited on Sept. 9)** **25 marks**

1. [1.5+1.5+3+4 = 10 marks] Consider a raised cosine (RC) pulse shaped transmission. The impulse response of the pulse-shaping continuous time filter g(t) is given as follows:

$$g(t) = Sinc(\frac{\pi t}{T}) \times \left(\frac{Cos(\frac{\pi \beta t}{T})}{1 - \frac{4\beta^2 t^2}{T^2}} \right)$$

where the "excess bandwidth" factor $0 \leq \beta \leq 1$ and T is the symbol duration and Sinc(x)=Sin(x)/x. We use a discrete-time filter g(nTs) = g(n) where the sampling rate Ts = T/J, where the oversampling factor J is a positive integer. Also, the filter is truncated to 2LT symbol durations, i.e., LT symbol durations on *each side* of t=0, where L is a positive integer. The input bit (or symbol) stream I(kT) into this filter will be zero-interleaved with J-1 zeros in order to create the output sequence x(kTs) which is sent into a DAC.

Choose your own N-bit (with N=32) random sequence where $I(k) \in \{+1, -1\}$ in order to create a BPSK signal. Assume all the bits before and after this 16-bit pattern are zeros. You are to plot the output sequence (samples) x(kTs) using both "symbol-plot" as well as join them using "line-plot" using Matlab.

(a) Plot as Fig.1, x(kTs) for J=4, L=2, and β=0.
(b) Repeat (a) and include in Fig.1 another plot with the change L=4.
(c) Plot as Fig.2, *three* different plots of x(kTs) where all of them have J=8, L=4, but with *three* different excess bandwidth factors, namely, β=0, β=0.5, and β=1. Comment on your results in (b) and (c).

(d) Plotting the PSD $S_X(f)$ using Periodogram: We now perform Monte-Carlo simulations and averaging to approximate the PSD of x(kTs). For this Periodogram approach, generate R=100 runs, each of bit-sequence length N=1024, for the RC pulse-shape with β=0.5, J=8, L=4. This would give in each MC run, 1024xJ = 8192 samples of x(kTs). Then, take a 8192-point FFT, square the magnitude response, and average it point-wise over the R=100 trials. Divide this by R (=100) to get an estimate of the PSD of the x(kTs), namely $S_X(f)$. Plot this as Fig.3. Now, also estimate in a similar manner the PSD of x(kTs) when the following "rectangular pulse-shape" is used on the bit-stream, namely: $g(t) = \begin{cases} \frac{1}{\sqrt{T}}, 0 < t \leq T \\ 0, \text{otherwise} \end{cases}$. Plot this PSD estimate using the periodogram approach, also on the same Fig.3, and comment.

2. [3+3+3+3+3 = 15 marks] In this question, we compare the theoretical average probability of symbol error P_E value and the computer simulated symbol error rate, SER, for three popular linear modulation schemes. The approximation on P_E using bounds will also be studied.

Consider the same energy per bit, E_B, for the 2-PAM (or BPSK), 4-QAM (or QPSK), and 16-QAM signals. Assume the complex baseband AWGN measurement model r(k)=I(k)+v(k), where i.i.d. symbols I(k) are uncorrelated with the white Gaussian noise v(k). For the two complex signals (4-QAM and 16-QAM), the noise will be complex with variance $\sigma^2 = N_O/2$ per dimension. Vary the signal to noise ratio, SNR, in the log-scale as $10\log_{10}(E_B / \sigma^2)$ in 2dB steps from 0dB to 14dB, by changing the noise variance. Chose E_B=1 for all the signal sets. The *Q*(.) function or the *erfc*(.) function can be evaluated using Matlab. *Hint:* Note that the energy per symbol will be different for each *M* and is given by $E_S = \log_2(M) E_B$ where *M*=2, 4, or 16, for the given three signal sets given above, respectively. Express "2*d*", the distance between the 2 nearest points in the constellation as described in the class, as a function of E_B, in order to compute "*q*". From *q*, get P_E.

(a) Compute and plot as Fig.4, the P_E for the 3 signals when sent thro the AWGN channel model. Plot the SNR against $\log_{10}(P_E)$ on the y-axis, for all the 3 signals.

(b) For the 4-QAM signal set, compute using the following approximations to P_E. Plot each of these values (bounds) on P_E for the same range of SNRs and call this Fig.5.

(b1) Union bound using *all* the pairwise symbol errors
(b2) Union bound using only the *nearest neighbours*
(b3) In part (b2) replace the *erfc*() function with the Chernoff bound and replot.
(b4) Plot also the accurate P_E obtained in part (a) also in Fig.5.
(b5) Comment on these results in Fig.5.

(c) Now, we wish to simulate the SER for 4-QAM (QPSK). For generating the transmit symbols, use uniform random variables between (0,1). If the r.v. takes a value between 0 and 0.5, it is mapped to say -*d*; else, it is mapped to +*d*, where 2*d* is the distance between the symbols. Note that $d = \sqrt{E_S} = \sqrt{2E_B} = \sqrt{2}$ in this case. We will need two uniform r.v.s, one of the real part and one for the imaginary part. For noise, generate 2 Gaussian r.v.s with appropriate variance, one for the real and one for the imaginary part. SNR is varies by changing the noise variance. (Explain your approach). Generate about 10^5 measurements r(k) to measure the SER over the same range of SNRs, i.e., 0dB to 14dB. Plot $\log_{10}$(SER) on the y-axis. Include this SER plot also into Fig.5 and comment.

(d) Repeat part (c) above for the 16-QAM signal set. Chose the uniform r.v. to symbol mapping appropriately. (Explain your approach). This SER plot should be called Fig.6. Plot the true P_E for 16-QAM found in part (a) also in Fig.6.

(e) Compute the union bound on P_E using *only the nearest neighbours* for the 16-QAM signal set. Plot this result also into Fig.6. Comment about all your results in Fig.6.

***Bonus Question -- 5 marks**:** Here, we are interested in computing the Bit Error Rate (BER) by using the mapping from symbols to bits. For a given SER, the BER can be minimized if Gray-mapping (coding) is done. Understand Gray mapping from the textbook and use this mapping for both 4-QAM and 16-QAM signals. (Explain your approach). Compute the BER after finding the SER, from all the symbols which were decoded in error. For the same range of SNRs as before, plot the following results in Fig.7. (i) SER of 4-QAM (ii) BER of 4-QAM (iii) SER of 16-QAM, and (iv) BER of 16-QAM. Comment about your results in Fig.7.

***** These 5 marks are over and above the 25 marks for the other questions.

Instructions

Submit only a hand-written report. Your name and roll-number must appear in the first page. All plots can be printed and attached to your report. Your working code must be properly commented and be emailed to the TAs before the due date. Their email address are: Sruti at ee18d705@smail.iitm.ac.in and Prasikaa at ee21d700@smail.iitm.ac.in. Your working code can be named "rollnumber-assignment1-code.m". Else, if you have not used Matlab, we can also accept any other convenient file format for seeing the results and the code. Python submissions are also okay. The TAs will get back to you if they require additional information. Please see other instructions, if any, in the WhatsApp group.