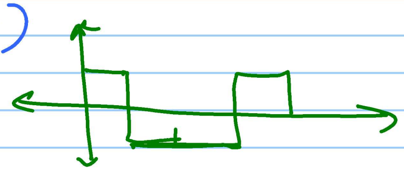
**Lesson 1a** -- Finding the basis signal set using Gram-Schmidt orthogonalisation

Note Title 8/8/2017

Given $M$ signals $\{s_1(t), s_2(t) \dots s_M(t)\}$ $(0,T)$ $\xrightarrow{\text{find}}$ $N$ basis functions $\{\phi_1(t), \phi_2(t) \dots \phi_N(t)\}$ $(0,T)$

$$\int \phi_j^2(t)\, dt = 1 \quad \forall j = 1 \dots N$$

$$\int \phi_j(t)\, \phi_k(t)\, dt = \begin{cases} 1, & j=k \\ 0, & j \neq k \end{cases}$$

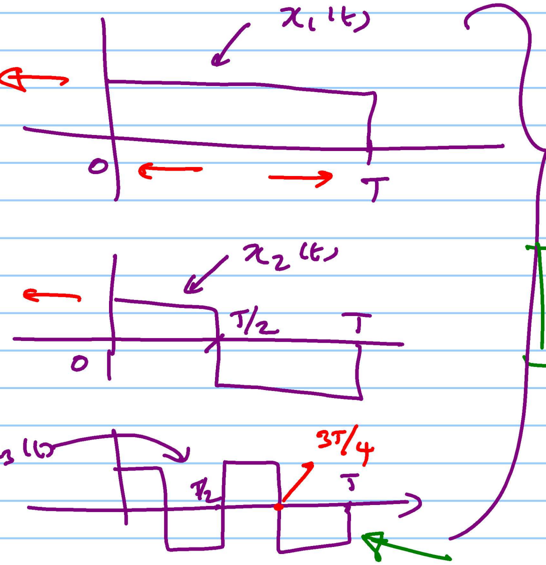


$2 \times \frac{1}{2T} \times T = 1$

$\frac{1}{2T}$

$\frac{1}{T}$

$\frac{2}{T}$




$\bar{V}_{3\times 1} = \text{l.c.}\{\vec{i}, \vec{j}, \vec{k}\}$

3 vectors

$\boxed{N = 2WT}$

$W \to$ Bandwidth

$2 \times \frac{4}{T}\, T \to 8$ fns.

Walsh-Hadamard sequences.

OVSF

Karhunen-Loeve

# Gram-Schmidt Procedure

$\{ s_1(t) \; \ldots \; s_M(t) \}$

$s_2(t)$ $\quad s_3(t)$

1. $\phi_1(t) = \dfrac{s_1(t)}{\sqrt{E_1}}, \quad E_1 = \int_0^T s_1^2(t)\,dt$

2. $\tilde{\phi}_2(t) = s_2(t) - s_{21}\,\phi_1(t)$ ← projection

   where $s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt$

3. $\phi_2(t) = \dfrac{\tilde{\phi}_2(t)}{\sqrt{E_2}}, \quad E_2 = \int_0^T \tilde{\phi}_2^2(t)\,dt$

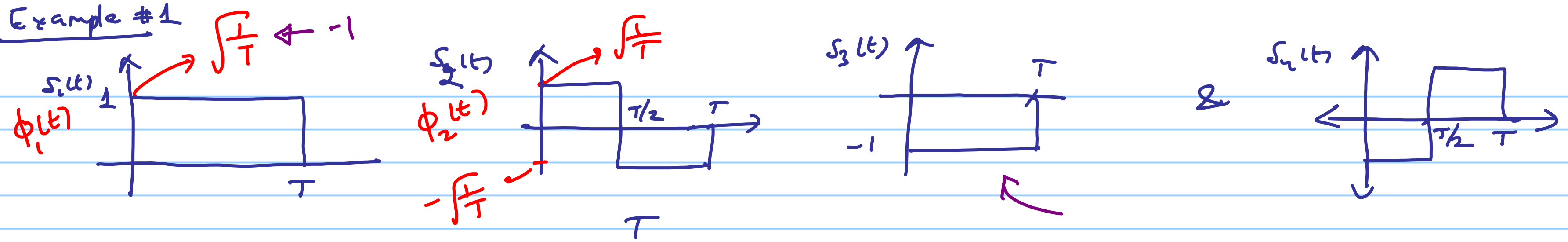
4. $\tilde{\phi}_3(t) = s_3(t) - s_{31}\,\phi_1(t) - s_{32}\,\phi_2(t)$

5. $\phi_3(t) = \tilde{\phi}_3(t) \,/\, \sqrt{E_3}, \quad E_3 = \int \tilde{\phi}_3^2(t)\,dt$

$s_{31} = \int_0^T s_3(t)\,\phi_1(t)\,dt$

$s_{32} = \int s_3(t)\,\phi_2(t)\,dt$

Example #1




1. $\phi_1(t) = \dfrac{s_1(t)}{\sqrt{E_1}}$ $\qquad E_1 = \int_0^T s_1^2(t)\,dt = T$

2. $\tilde{\phi}_2(t) = s_2(t) - s_{21}\,\phi_1(t)\,, \quad s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt = 0$

$\Rightarrow \tilde{\phi}_2(t) = s_2(t)$

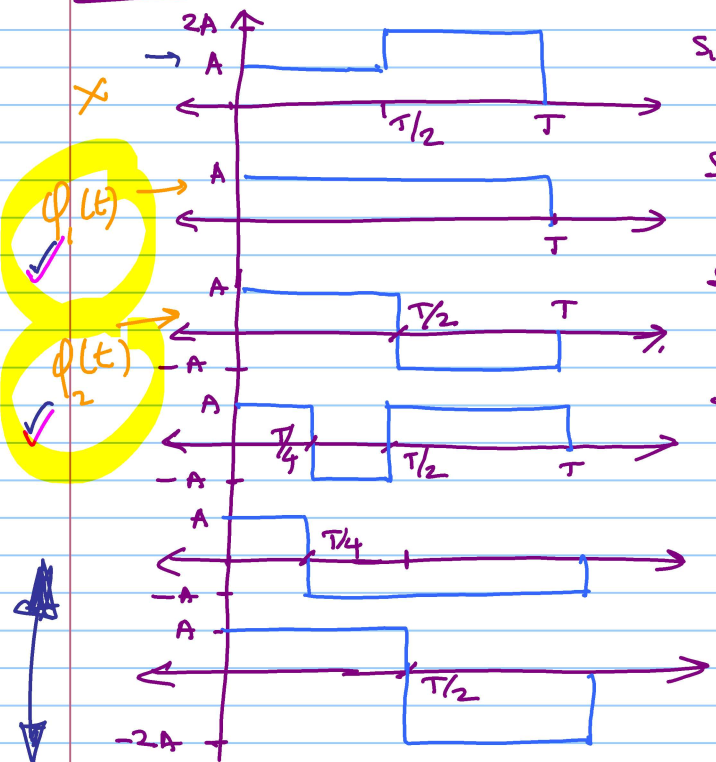
3. $\phi_2(t) = \dfrac{\tilde{\phi}_2(t)}{\sqrt{E_2}} = \dfrac{s_2(t)}{\sqrt{T}}$ $\qquad s_{31} = \int_0^T s_3(t)\,\phi_1(t)\,dt = -\sqrt{T}$ ✓ ;

$\frac{1}{\sqrt{T}}\int_0^T -1\,dt$

4. $\tilde{\phi}_3(t) = \underbrace{s_3(t) - s_{31}\,\phi_1(t)}_{} - s_{32}\,\phi_2(t)$ $\qquad s_{32} = 0$ ;

$= 0$ ; $\qquad |||^{ly}\ \tilde{\phi}_4(t) = 0$

## Example #2 :




(a) Find the basis set for this $\{s_i(t)\}$ $i = 1 \ldots 6$

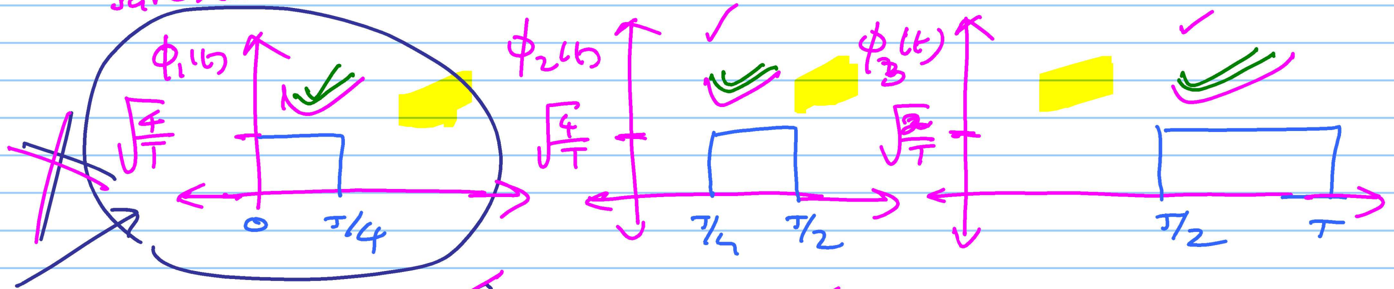
Vector repr. $\bar{s}_1, \bar{s}_2 \ldots \bar{s}_6$

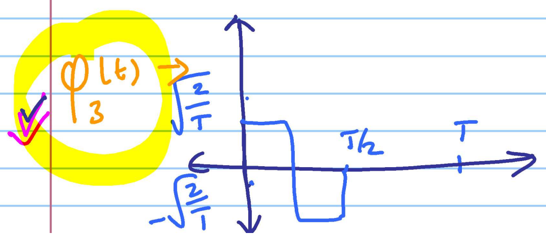
$\boxed{N = 2WT}$

(b) Plot the signal constellation

(c) Develop the "most efficient" Matched filter receiver for this signal set.

Surendra's Basis



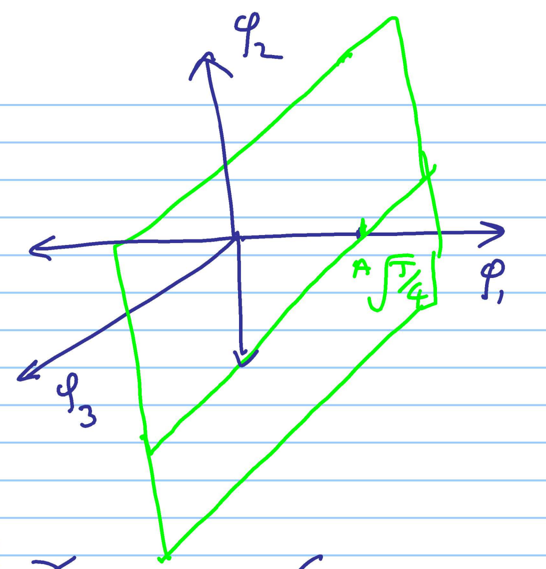




$\phi_1(t) = \frac{s_2(t)}{\sqrt{E_2}}$

$s_3(t) \rightarrow \phi_2(t)$

$\bar{S}_1 = \begin{bmatrix} A\sqrt{\frac{T}{4}} \\ A\sqrt{\frac{T}{4}} \\ 2A\sqrt{\frac{T}{2}} \end{bmatrix}$ ; $\bar{S}_2 = \begin{bmatrix} A\sqrt{\frac{T}{4}} \\ A\sqrt{\frac{T}{4}} \\ A\sqrt{\frac{T}{2}} \end{bmatrix}$ ; $\bar{S}_4 = \begin{bmatrix} A\sqrt{\frac{T}{4}} \\ -A\sqrt{\frac{T}{4}} \\ A\sqrt{\frac{T}{2}} \end{bmatrix}$ ......

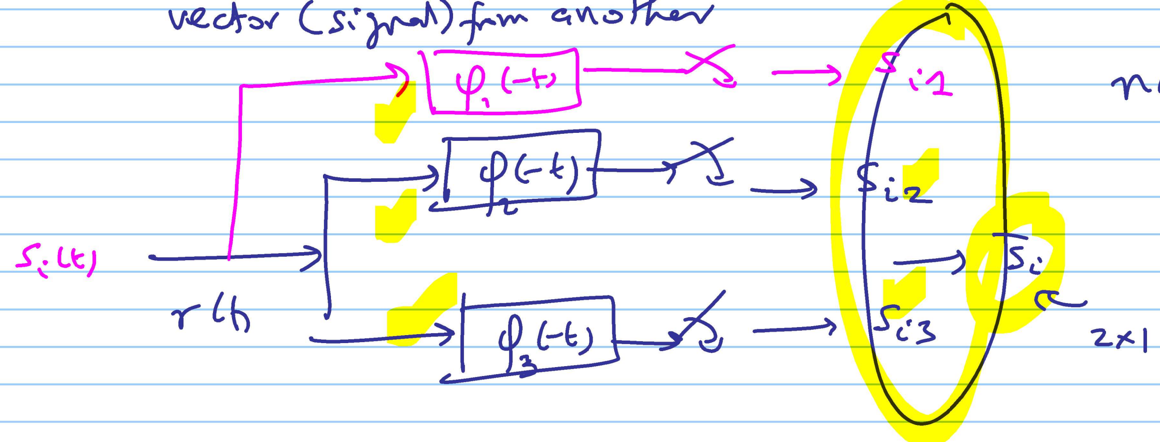
$$\bar{s}_3 = \begin{bmatrix} A\sqrt{\frac{T}{4}} \\ A\sqrt{\frac{T}{4}} \\ -A\sqrt{\frac{T}{2}} \end{bmatrix} ; \quad \bar{s}_5 = \begin{bmatrix} A\sqrt{\frac{T}{4}} \\ -A\sqrt{\frac{T}{4}} \\ -A\sqrt{\frac{T}{2}} \end{bmatrix} ; \quad \bar{s}_6 = \begin{bmatrix} A\sqrt{\frac{T}{4}} \\ A\sqrt{T/4} \\ -2A\sqrt{T/2} \end{bmatrix} ;$$

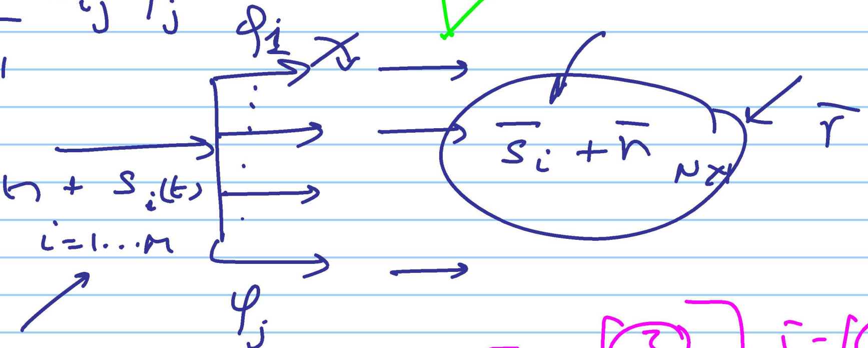



⇒ Since $s_{i1}$ is common in all $\bar{s}_i$

⇒ It does not add to the discrimination of one vector (signal) from another

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\, \phi_j(t) \qquad i = 1 \dots M$$




$$\bar{a} = \begin{bmatrix} 3 \\ 0.1 \\ 0.7 \end{bmatrix} \quad \bar{b} = \begin{bmatrix} 3 \\ 0.2 \\ 0.1 \end{bmatrix}$$

$$(\bar{a} - \bar{b})^T (\bar{a} - \bar{b})$$

$$\underset{3\times1}{\bar{r}} = \underset{3\times1}{\bar{s}_i} + \underset{3\times1}{\bar{n}}$$

Pick $\hat{s} = s_i$

iff $\boxed{(\bar{r} - \underset{3\times1}{\bar{s}_i})^T (\bar{r} - \bar{s}_i) \leq (\bar{r} - \bar{s}_l)^T (\bar{r} - \bar{s}_l) \quad \forall\, l \neq i, \; l = 1 \dots M}$

$\bar{s}_i$ 2x1

Minimum Distance Rule

$$\|s_i(t)\| = \left( \int_0^T s_i^2(t)\, dt \right)^{1/2}$$

$$s_i(t) \longleftrightarrow \bar{s}_i \quad N\times1$$

$i = 1 \dots M$

Triangle Inequality

$$\| s_1(t) + s_2(t) \| \leq \| s_1(t) \| + \| s_2(t) \|$$

$$\| \bar{s}_1 + \bar{s}_2 \| \leq \| \bar{s}_1 \| + \| \bar{s}_2 \|$$

$$\left| \int_a^b x_1(t)\, x_2^*(t)\, dt \right| \leq \left( \int_a^b |x_1(t)|^2 dt \right)^{1/2} \cdot \left( \int_a^b |x_2(t)|^2 dt \right)^{1/2}$$

Cauchy-Schwartz Inequality

$$\| v_1^H v_2 \| \leq \| v_1 \| \cdot \| v_2 \|$$