Department of Electrical Engineering
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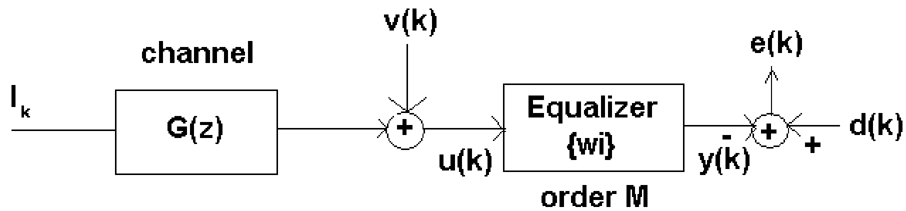
EE 4140: Digital Communication Systems

November 25 2020 **Tutorial #4** KG/IITM

1. In the figure below, the input $\{I_k\}$ is i.i.d with $E[I_k^2]=1$ and $P[I_k=+1]=1/2= P[I_k=-1]$. The AWGN v(k) has variance $\sigma_v^2=0.2$ and $E[I_k\, v(i)]=0$ for all k,i.



The channel $G(z)=1-z^{-1}+0.5\,z^{-2}$ and the equalizer has an order equal to M. For the following situations, compute manually $\mathbf{R}$, $\mathbf{p}$, and eventually $\mathbf{w}_{opt}=\mathbf{R}^{-1}\mathbf{p}$.

a) M=2, d(k)=I(k)
b) M=2, d(k)=I(k-1)
c) M=2, d(k)=I(k-2)
d) What is the J_{min} in each of the above cases?
e) Now, consider a Decision Feedback Equaliser (DFE) with M=2 taps in the feedforward section and 2 taps in the feedback section. For each of the cases in (a), (b), and (c), what will be the resultant **R** and **p** in this case?

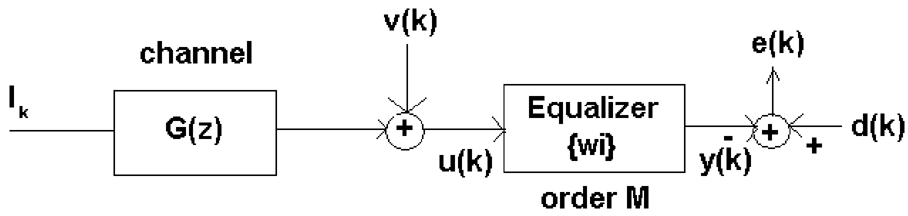
2. An uniform iid sequence $I(n)\in\{-1,+1\}$ is transmitted through a FIR channel $H(z)=1-2z^{-1}+0.5z^{-2}$ and the resultant output *x(n)* is corrupted by an AWGN sequence *u(n)* with variance $\sigma_u^2=0.3$. It is required to define a 2-tap linear equalizer to filter the measurement samples *z(n) = x(n)+u(n)*. Assume that {*I(n)*} and {*u(n)*} are mutually uncorrelated.

(a) Specify the auto-correlation matrix clearly.
(b) If the desired sequence is defined by *d(n)= I(n-Δ)*, find the 2-tap linear MMSE equalizer with the least value for Jmin by varying Δ. What is this "best" value for Δ ?
(c) What is the corresponding Jmin for this LE-MMSE?
(d) What is the variance of the residual ISI contribution for this LE-MMSE? If this excess ISI is considered as an an additional, uncorrelated, Gaussian noise component, what will be the expression for the average probability of symbol error at the output of the equalizer? (Write this in terms of the $q(d)$ where $q(d)=Q\left(\dfrac{d}{\sqrt{N_0/2}}\right)$ with $2d$ as the distance between the symbols.)

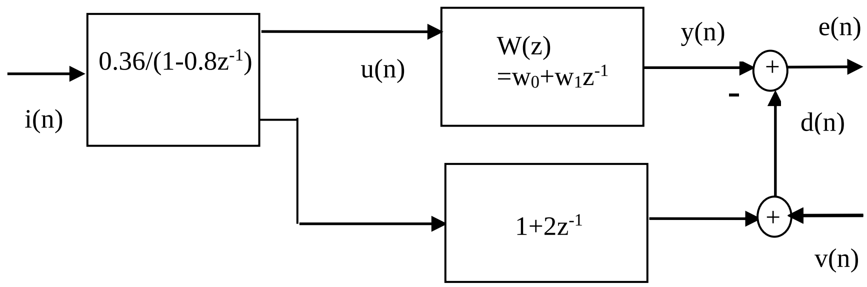
3. A uniform real i.i.d sequence {d[k]} with $E\{|d[k]|^2\}=1$ is filtered by $H(z)= 1-0.5z^{-1}+(1/3)\,z^{-3}$ and the resulting output is corrupted by a coloured noise which is a result of AWGN filtered by $1+0.8z^{-1}$ to give the measurements {u[k]} where the variance of the AWGN is 0.25 and this AWGN is uncorrelated with {d[k]}.

a) Find $\mathbf{R}_{uu}$ of size 3×3.
b) Find a 3×1 $\mathbf{p}=E\{\mathbf{u}[k]d[k-\Delta]\}$ for (i)Δ=1, (ii)Δ=4.

4. In the figure below, the input $\{I_k\}$ is i.i.d with $E[I_k^2]=1$ and $P[I_k=+1]=1/2= P[I_k=-1]$. The AWGN v(k) has variance $\sigma_v^2=1$ and $E[I_k v(i)]=0$ for all k,I, and the channel response $G(z)=1/(1-0.8z^{-1})$. If **w** is of order M=2, find $\mathbf{w_o}$ for (a) $\sigma_v^2=1$, (b) $\sigma_v^2=0.3$ and (c) $\sigma_v^2=0$.



5. Consider the measurement model below, where a linear MSE (Wiener) estimator is to be defined for the equalizer W(z) which will minimize $E[e^2(n)]$. Here, input i(n) is white noise with unit variance, and additive noise v(n) has variance $\sigma_v^2 = 0.10$ and is uncorrelated with i(n).



(a) Find the correlation matrix **R** and the cross-correlation vector **p**.
(b) What is the LMSE estimate for W(z)?
(c) What is the Jmin for this LMSE problem?

6. For the measurement model below, a decision feedback equalizer w with <u>3 feed-forward</u> taps and <u>2 feedback taps</u> is to be defined so as to minimize $E[e^2(k)]$. Here, input symbols $i(n)\epsilon\{+1,-1\}$ are equi-probable, and the additive noise v(k) has variance $\sigma_v^2 = 0.10$ and is uncorrelated with i(n). If the channel $F(z)=1+2z^{-1}-z^{-3}$, and the decoding delay $\Delta=1$, specify the entries of: (a) the auto-correlation matrix **R**, and (b) the cross-correlation vector **p**.