Department of Electrical Engineering
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EE 4140: Digital Communication Systems

November 24 2020 **Tutorial #3** KG/IITM

1. Consider the "square" 4-QAM (set x), 16-QAM (set y), and 64-QAM (set z) signal sets.
(a) Plot the 16-QAM signal constellation with "Gray Coding" to ensure that all the nearest neighbor symbols differ only by 1-bit labels.
(b) For the *same* average energy per bit, E_b, find the minimum distances d_x of 4-QAM and d_y of 16-QAM in terms of the minimum distance d_z of the 64-QAM constellation.
(c) What is the accurate expression for the average symbol error probability in all the 3 cases?
(d) If $E_b/No = 10$ for all the 3 signals, using the Chernoff bound on the Q-function, calculate the numerical value of the probability of symbol error for the 3 signals.
(e) Reconciling your answers in (a) and (d), what will be the (numerical) value of the bit error probability?

2. Again, recall this problem from tutorial #2, where the signal constellation in Fig. 1 with minimum distance 2*d* is used When this signal is sent through an ideal channel and corrupted by additive white Gaussian noise with variance No/2, and after matched filtering and sampling, the received samples are given by $r(k) = s_i(k) + n(k)$ where $i \in \{1,2,...,6\}$.

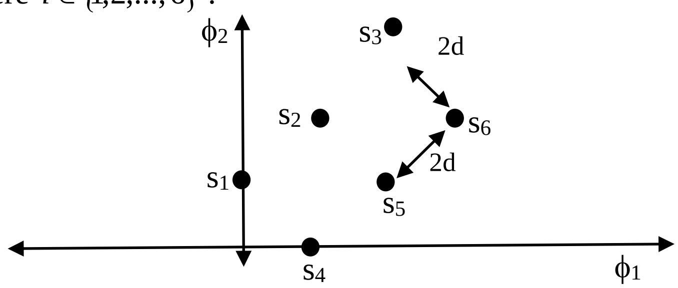

Fig. 1

(a) Assuming that all the symbols are equi-probable, find the exact expression for the average probability of symbol error P_e in the above AWGN channel. Express your answer in terms of $q(d) = Q\left(\dfrac{d}{\sqrt{N_0/2}}\right)$ with 2*d* as the minimum distance.
(b) Now instead, let the probability of occurrence of symbols s_1 and s_5 is 1/3, while that of the remaining 4 symbols is 1/12 each. Make a rough plot of the new decision regions, if any.
(c) Can you now find the exact expression for P_e now? Else, use the Union-Bound argument to get an (approximate) expression for P_e. Explain the assumptions made.

3. Consider the two 8-ary signal constellations in Fig. 2, each with minimum distance 2*d*. When either of this signal is sent through an ideal channel and corrupted by additive white Gaussian noise with variance $N_0/2$, and after matched filtering and sampling, the received samples are given by $r(k) = s_i(k) + n(k)$ where $i \in \{1,2,...,8\}$.

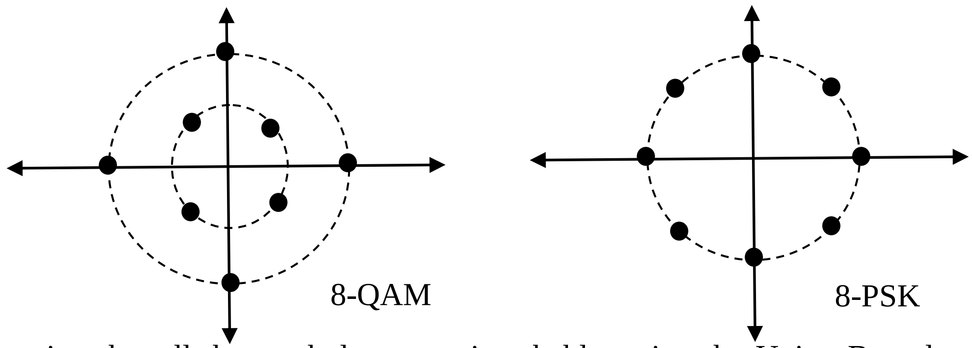

Fig. 2

(a) Assuming that all the symbols are equi-probable, using the Union Bound only over the nearest neighbor(s), find the expression for the approximate average probability of symbol error P_e in the above AWGN channel for the 8-PSK signal set. Express your answer in terms of $q(d) = Q\left(\frac{d}{\sqrt{N_0/2}}\right)$ with 2*d* as the minimum distance.

(b) Repeat (a) for the circular 8-QAM signal set.

For the same energy per bit, which of the 2 signal sets will have the lower approx. P_e ? Why? Explain.

4. In a low-pass channel with one-sided bandwidth of 2MHz, sinc-shaped QPSK symbols are transmitted.

(a) What is the bit-rate of this link?

(b) If excess bandwidth factors of either β=0.5 or β=1.0 are to be used along with SRRC pulse-shaping, what modulations must be used in each of these 2 choices in order to preserve the same bit-rate?

5. Consider a received signal $z(n) = \sum_{l=0}^{L-1} c_l \, I(n-l) + w(n)$, where the *L*=6 tap FIR channel has complex coefficients c_0 to c_5, with $c_2 = 0+j0$. The complex data *I(n)* is drawn from the QPSK alphabet and the noise *w*(*n*) is AWGN with variance σ_w^2. The Viterbi Algorithm (VA) is to be used to implement MLSE for this measurement model.

(a) How many states (nodes) will the VA have?

(b) How many transition metric computations will be required at every symbol time *nT*?

(c) Assume that the "all 1+j1" sequence was transmitted over 10 symbol periods (*n*=1 thro 10). Consider all those sequences that emerge from this sequence and re-merge with this "all 1+j1" sequence in the least number of hops (i.e., over the least number of branches). How many such sequences are there? Can you enumerate these sequences of length 10, if only one emergence and one mergence from the "all 1+j1" within these 10 symbol periods is to be considered? *Hint*: You can do this without drawing the full trellis diagram over the 10 symbol periods!

6. The received signal thro' a 2-tap channel is given by $z(n) = \sum_{l=0}^{1} f_l\, I(n-l) + v(n)$, where the FIR channel coefficients f_0= 0.9, and f_1=0.4, and data *I(n)* and noise *v*(*n*) are mutually uncorrelated with $I(n) \in \{-1,+1\}$ and the noise is AWGN with variance σ_v^2 = 0.3. The Viterbi algorithm is used to implement MLSE for this measurement model.

(a) Draw a single-stage of the VA, clearly labeling the nodes, and the branches.

(b) The first six values of *z(n)* are given as follows: z(1) = -1.1; z(2) = 0.4; z(3) = 1.5; z(4) = 1.2; z(5) = -0.6; and, z(6) = -1.2. Assuming that $I(n) = -1,\ n \le 0$, compute the evolution of the VA over the 6 time-intervals. Indicate the values of the Cumulative Metrics (of all the nodes) at the end of time *n*=6.

(c) What is the ML sequence as indicated by the VA at the end of time *n*=6?

7. Do the following problems from the 8th chapter in the text-book (Proakis and Salehi), starting with page. 561 (pdf #290) in the E-version. The problems marked with "*" could be a little bit tougher.

➔ Problems from **8.1 to 8.3** , plus **8.9 to 8.12** , plus **8.14 to 8.16** , plus **8.18 to 8.21** , and **8.52*** .