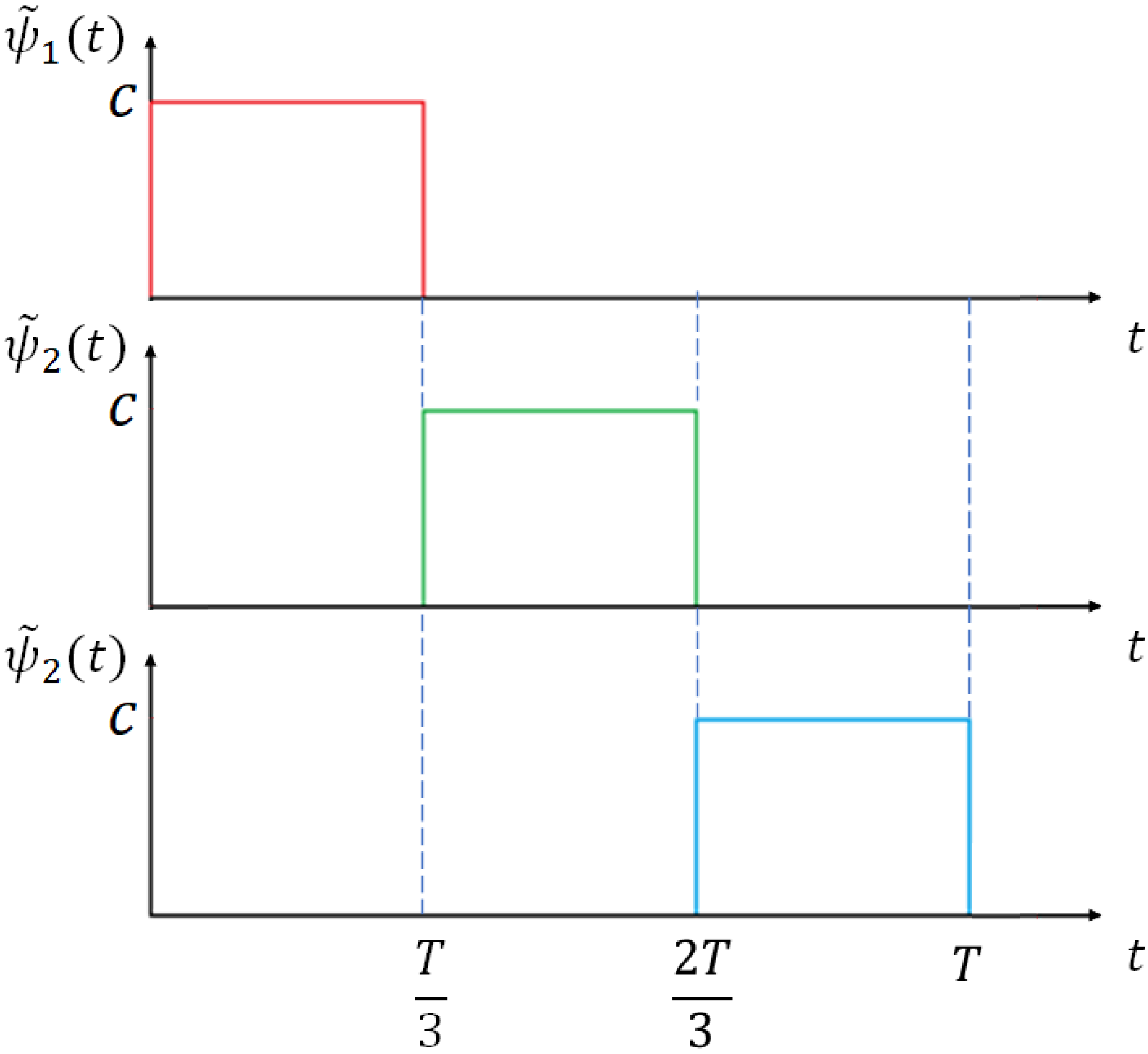
Compact basis set

For Q5, consider the following set of waveforms as the basis set.



Now,

$$\begin{pmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \\ s_4(t) \\ s_5(t) \\ s_6(t) \end{pmatrix} = \frac{1}{c} \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 2 & 0 & 0 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \\ -2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{\psi}_1(t) \\ \tilde{\psi}_2(t) \\ \tilde{\psi}_3(t) \end{pmatrix}, \quad where \quad c = \sqrt{\frac{3}{T}}.$$

Let $A = \frac{1}{c} \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 2 & 0 & 0 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \\ -2 & 0 & 0 \end{pmatrix}.$

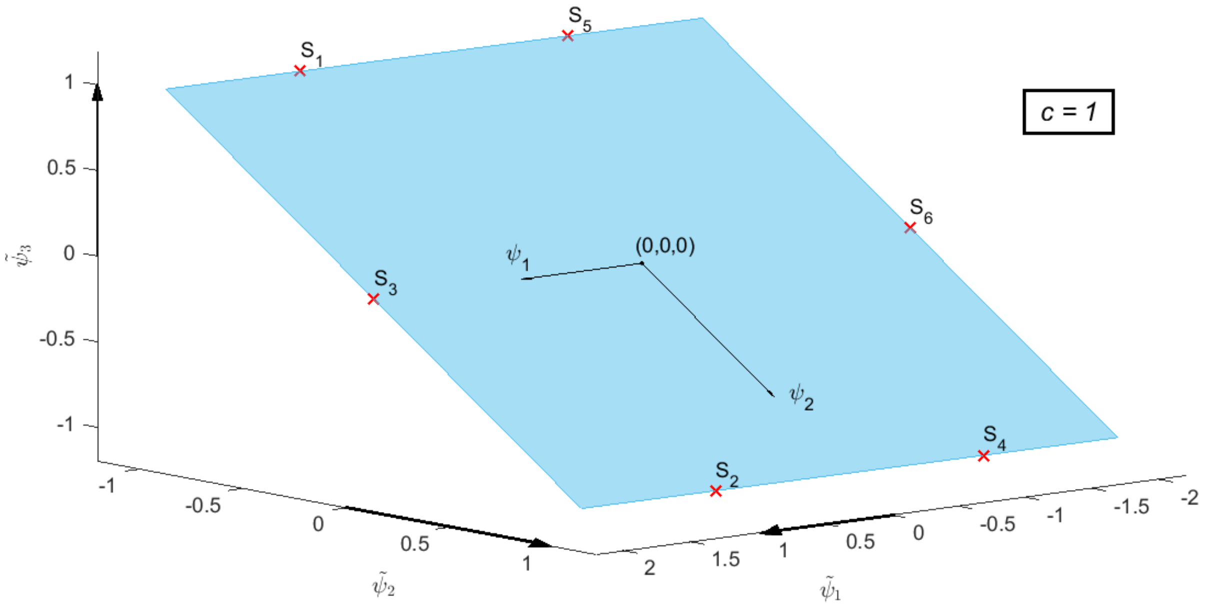
The rows of A are the representations of the signal in space spanned by $\tilde{\psi}_i(t),\ i = 1, 2$ and 3.

$$rank(A) = rank\left[\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 2 & 0 & 0 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \\ -2 & 0 & 0 \end{pmatrix}\right] = rank\left[\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

$$= rank\left[\begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right] = rank\left[\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

$$= rank\left[\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right] = \mathbf{2}.$$

Thus, the vectors representating the signals lie in a 2-dimensional space within the 3-dimensional space spanned by $\tilde{\psi}_i(t), i = 1, 2$ and 3. This is illustrated in the following figure.



Compact description of the signals is possible by only considering 2 orthonormal bases instead of 3.

Let

$$\psi_1(t) = \tilde{\psi}_1(t) \qquad and \qquad \psi_2(t) = \frac{1}{\sqrt{2}}\left(\tilde{\psi}_2(t) - \tilde{\psi}_3(t)\right).$$

These two are the bases considered in the solution.