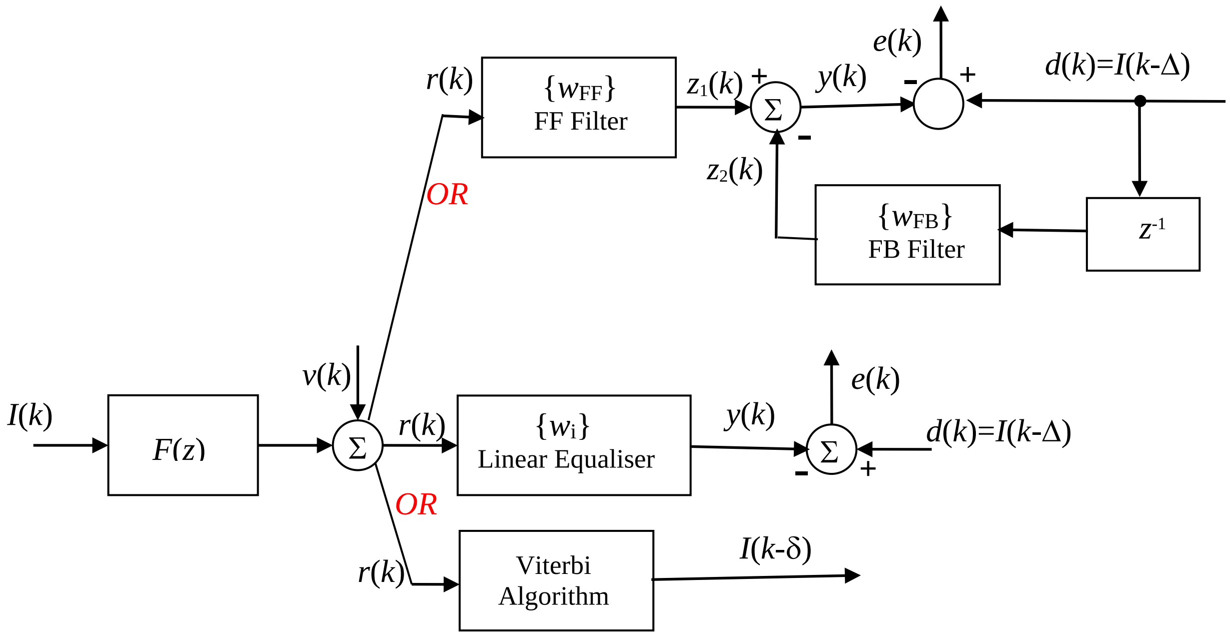
Department of Electrical Engineering, IIT Madras
EE-4140: Digital Communication Systems

Due by 5pm, Dec 21, 2020

December 2020 **Simulation Assignment #2** **25 marks**

Consider a distorting (ISI) channel defining the measurement model as shown in the figure below. Here, the independent, uniformly distributed data symbols $I(k)$ entering the channel $F(z)$ are drawn from a 4-ary PAM alphabet, and the transmit signal power $E[|I(k)|^2] = \sigma_I^2 = 1$. The 3-tap channel is specified by the transfer function $F(z) = \frac{1}{\sqrt{2}}(0.8 - z^{-1} + 0.6z^{-2})$ and the additive white, Gaussian, noise component $v(k)$ is zero mean with variance σ_v^2. The signal to noise ratio (SNR) measured on $r(k)$, is then given by $1/\sigma_v^2$ (why?), and we vary the SNR on the measurements by varying σ_v^2 appropriately. Either a MLSE based approach using the Viterbi Algorithm (VA), *OR*, a N^{th}-order linear equalizer (LE), *OR*, a decision feedback equalizer (DFE) with N_1 feedforward (FF) taps and N_2 feedback (FB) taps where $N_1+N_2=N$, is used to mitigate the ISI induced distortion on the measurements.



(a) [10 marks] Define a VA based sequence estimator, assuming perfect channel information is available at the receiver. Generate for each SNR (in 2dB steps from 0dB to 16dB), say 100,002 symbols of 4-PAM using uniform random variables appropriately to get $I(k)$. The noise $v(k)$ is generated using the normal pdf, and the value returned is scaled using the current σ_v (for the given SNR). The last 2 symbols of $I(k)$ may be viewed as "tail symbols" which are known to the receiver, in order to terminate the VA in a known state. Measure SER only over the (unknown) remaining 100,000 symbols, for the following choices of decoding delay δ, namely: (a1) δ=3; (a2) δ=6; (a3) δ=15; and (a4) δ=30; Compute the $\log_{10}$SER versus $10\log_{10}$(SNR) for each choice of δ, and plot all the four results on the same Fig 1.

(b) [6 marks] In the following, we study the performance of a linear equaliser (LE) for the above channel at SNR=10dB.

(b1) The LE has order N=3, and the desired response $d(k)=I(k-\Delta)$, where the decoding delay Δ=0. Determine the LMSE (Wiener) solution $\mathbf{w}_{opt}$. What is the corresponding J_{min}?

(b2) Repeat (b1) for N=10 and Δ=0.
(b3) Repeat (b1) for N=10 and Δ=5.
(b4) Can you find the "best choice of N and Δ" for SNR=10dB by trial-and-error, to give the lowest possible J_{min}? What is this value of J_{min}?
(b5) For SNR in 2dB steps as in part **(a)**, evaluate and plot $\log_{10}$SER versus $10\log_{10}$(SNR) for your best choice of N and Δ, determined for the LE from part (b4). Mark this as Fig 2. Measure SER over the 100,000 4-PAM symbols as in part (a). In this Fig 2, include the earlier SER plot from (a4), i.e., VA performance for δ=30. Comment on your result.

(c) [9 marks] Finally, we study the performance of a decision feedback equaliser (DFE) for the same channel at SNR=10dB. The feedforward filter of order N_1 produces the output $z_1(k)$ and from this, $z_2(k)$, the output of the feedback filter (of order N_2) is subtracted to produce the symbol estimate $y(k)$.

(c1) The DFE has order N_1=6 and N_2=4, and the desired response $d(k)=I(k-\Delta)$, where the decoding delay Δ=0. Determine the 10x1 LMSE (Wiener) solution $\mathbf{w}_{opt} = [\mathbf{w}_{ff} \mid \mathbf{w}_{fb}]$. What is the corresponding J_{min}?
(c2) Repeat (b1) for Δ=3.
(c3) For what other choice of N_1+N_2 = 10 can you obtain the lowest possible J_{min}? What is this value of J_{min}, and the corresponding values of N_1, N_2, and Δ ?
(c4) For SNR in 2dB steps as in part **(a)**, evaluate and plot $\log_{10}$SER versus $10\log_{10}$(SNR) for your best choice of N and Δ, determined for the DFE from part (c3). Measure SER over the 100,000 4-PAM symbols as in part (a). Plot this also in Fig 2 and comment on your result.

Bonus Question (2 marks): For you best choice of 10-tap LE and 10-tap DFE, at SNR=10dB, what will be the (sum) variance of the residual inter-symbol interference terms? Show how you computed this in each case. *Hint*: Consider the "effective" channel after applying the equaliser.

Instructions: Submit only a single PDF file for the report. Name the file as "rollnumber-assignment2-report.pdf". Your working code, properly commented, **must be included** as Annexure-1 to the PDF file, and also separately sent by email to the TAs. Your working code can be named "rollnumber-assignment2-code.m". Else, we can also accept any other convenient file format for seeing the results and the code. Python submissions are also okay. The TAs will get back to you if they require additional information. Please see other instructions, if any, in the WhatsApp group.