

Oct. 21, 2020

Simulation Assignment #1

20 marks

1. [5 marks] We consider raised cosine (RC) pulse shaped transmission. The impulse response of the pulse-shaping continuous time filter $g(t)$ is given as follows:

$$g(t) = \text{Sinc}(\pi t/T) \{ \text{Cos}(\pi\beta t/T) / (1-4\beta^2 t^2/T^2) \}$$

where the excess bandwidth factor $0 \leq \beta \leq 1$ and T is the symbol duration and $\text{Sinc}(x) = \text{Sin}(x)/x$. We use a discrete-time filter $g(nT_s) = g(n)$ where the sampling rate $T_s = T/J$, where J is a positive integer. Also, the filter is truncated to $2LT$ symbol durations, i.e., LT symbol durations on each side of $t=0$, where L is a positive integer. The input bit (or symbol) stream $I(kT)$ into this filter will be zero-interleaved with J zeros in order to create the output sequence $x(kT_s)$ which is then sent into a DAC.

Chose your own 16-bit random sequence where $I(k) \in \{+1, -1\}$ in order to create a BPSK signal. Assume all the bits before and after this 16-bit pattern are zeros. You are to plot the output sequence (samples) $x(kT_s)$ using both “symbol-plot” as well as join them using “line-plot” using Matlab.

- Plot as Fig.1, $x(kT_s)$ for $J=4$, $L=2$, and $\beta=0$.
- Repeat (a) and include in Fig.1 another plot with the change $L=4$.
- Plot as Fig.2, *three* different plots of $x(kT_s)$ where all of them have $J=8$, $L=2$, but with *three* different excess bandwidth factors, namely, $\beta=0$, $\beta=0.5$, and $\beta=1$.
- Comment on your results in (b) and (c).

2. [15 marks] In this question, we compare the theoretical probability of symbol error $P(e) = P_S$ and/or the computer simulated symbol or bit error rate P_B (SER or BER), for a few popular linear modulation schemes. The approximation on P_S using bounds will also be studied.

(a) For the same energy per bit, E_b , compute and plot on the same figure (Fig-1) the P_S of BPSK, QPSK, and 16-QAM signals. Assume the complex base-band AWGN measurement model $r(k) = I(k) + v(k)$, where iid $I(k)$ is uncorrelated with the white Gaussian noise $v(k)$. For complex signals (QPSK and 16-QAM) the noise will be complex with variance $N_0/2$ per dimension. Vary the ratio $10\log_{10}(E_b/N_0)$ in 2dB steps from 0dB to 10dB, by changing the noise variance and plot against $\log_{10}(P_S)$, for all the 3 signals. Chose $E_b=1$ for all the signal sets. Note that the log of the error probability is plotted on the y-axis. The $Q(\cdot)$ function or the $\text{erfc}(\cdot)$ function can be evaluated using Matlab.

(b) For the QPSK signal set, compute using the following approximations to the symbol error probability. Plot each of these values (bounds) of P_S for the range of SNRs from 0dB to 10dB in the same figure. Call it Fig-2. Plot the true P_S obtained in part (a) also in Fig-2.

- Union bound using *all* the pairwise symbol errors
- Union bound using only the *nearest neighbours*
- In part (b2) replace the $\text{erfc}(\cdot)$ function with the Chernoff bound and add this also to Fig-2.

(c) Now, we wish to simulate the symbol error rate (SER) P_B for QPSK. For generating the transmit symbols, use uniform random variable (r.v.) between (0,1). If the rv takes a value X between 0 and 0.5, it is mapped to say $-d$; else, it is mapped to $+d$, where $2d$ is the distance between the symbols. Note that

$d=\sqrt{E_b} = 1$ in this case. We will need two r.v.s, one of the real part and one for the imaginary part. Generate about 10^5 symbols to measure the SER over the same range of SNRs as in part (c). Plot your results for $\log_{10}(P_B)$ also into Fig-2. Comment.

(d) Repeat part (c) above for the 16-QAM signal set. Call this SER result as Fig-3. Chose the uniform r.v. to symbol mapping appropriately and explain how you did it.

(e) Repeat parts (b2) and (b3) for the 16-QAM signal set to get the bounds. Note that the full Union bound in (b1) is not required. Plot these results also in Fig-3.

(f) With and without Gray-mapping (coding) done, compare the simulated bit error rate P_{BER} of QPSK and 16-QAM. Plot these four results together in Fig-4, again for $E_b=1$ in both cases. Note that for computing BER, you will have to make symbol decisions, and de-map them to bits and then compute P_{BER} . The de-mapping (and mapping scheme) with and without Gray-coding will be different.

Instructions: Submit only a single PDF file for the report. Name the file as “rollnumber-assignment1-report.pdf”. Your working code, properly commented, **must be included** as Annexure-1 to the PDF file, and also separately sent by email to the TAs. Your working code can be named “rollnumber-assignment1-code.m”. Else, we can also accept any other convenient file format for seeing the results and the code. Python submissions are also okay. The TAs will get back to you if they require additional information. Please see other instructions, if any, in the WhatsApp group.