EE4140 **Class # 9** - September 16, 2020

Note Title 3/27/2020

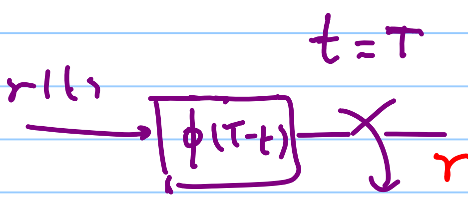
M-ary

Optimum Decision Rule → For the Receiver to decode the symbol

$$\vec{r}(t) = s_m(t) + n(t), \quad m = 1 \ldots M \leftarrow 2^K$$

(N)

$\{\phi_j(t)\}$ → $j = 1 \ldots N$

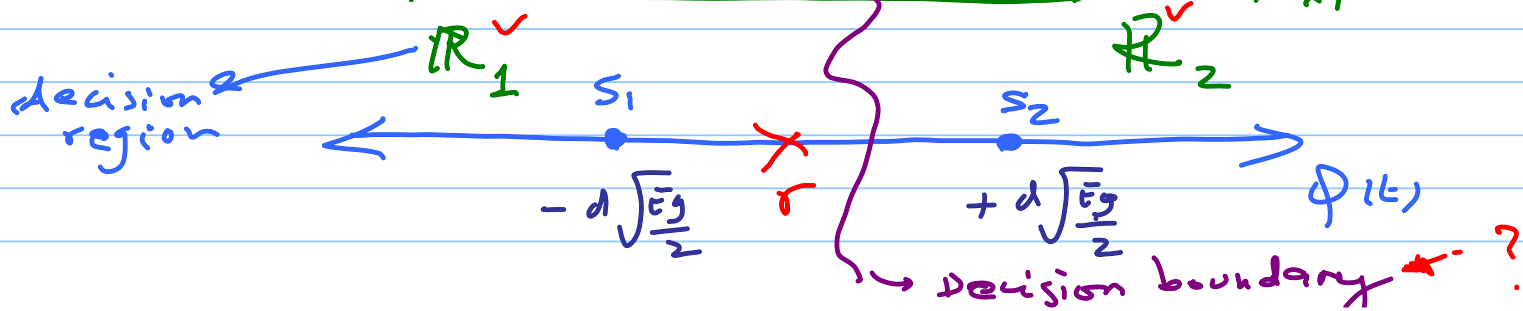


$$\boxed{r = s_m + n}$$

N = 1

N×1

$$s_m \in \pm \frac{d\sqrt{E_g}}{2}$$



Average Probability of Correct (Symbol) Decision
Bit

$$P(c) = \sum_{m=1}^{2} \underbrace{P(c|s_m)}_{= \int_{R_m} p(r|s_m)\, dr} P(s_m)$$

$$= \sum_{m} \int_{R_m} \underbrace{p(r|s_m)\, p(s_m)}\, dr$$

$p(s_m) = \frac{1}{M}$

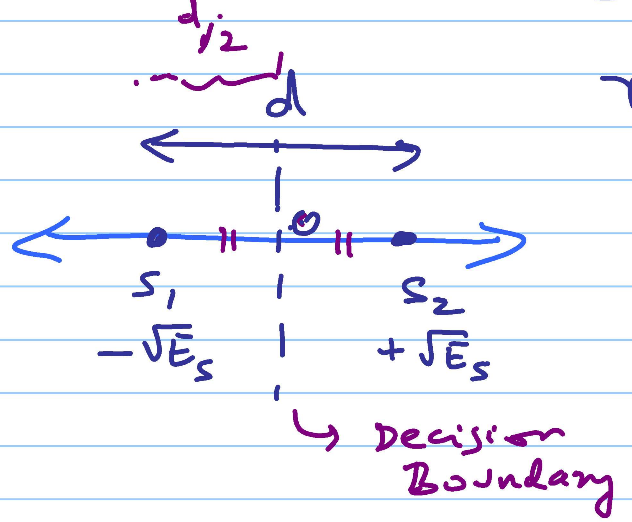
Maximize $P(c)$

$\Longrightarrow \max\ p(r|s_m)\, p(s_m) \quad \forall\ s_m$

$\Longrightarrow \max\ p(s_m|r)\, p(r)$ → ≠ fn. s_m

$\Longrightarrow \max\ p(s_m|r) \quad \forall\ s_m$ → Maximum A Posteriori (MAP) Rule

Probability of Symbol Error



$$r = s_m + n \qquad m = 1, 2$$

$n \sim \mathcal{N}\left(0, \frac{N_0}{2}\right) \rightarrow \sigma^2$

$P(s_1) = P(s_2) = \frac{1}{2}$;

$\max P(c) \implies$ MAP rule $\max_{s_m} p(s_m | r)$

$\implies \max_{s_m} \underbrace{p(r | s_m)}_{\text{likelihood}} p(s_m)$

(ML) Maximum Likelihood Rule $\implies \max_{s_m} p(r | s_m)$

mixed r.v.

Discrete r.v.

cont. r.v.

$$\underbrace{r = s_m + n}$$

$$n \rightarrow \mathcal{N}\left(0, \frac{N_0}{2}\right)$$

$$\frac{1}{\sqrt{\pi N_0}} e^{-\frac{n^2}{\frac{N_0}{2} \times 2}}$$

Assuming $\frac{N_0}{2}$ is known.

$$p(r|s_m) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-s_m)^2}{N_0}}$$

pick $\hat{s} = s_1$ iff $\boxed{p(r|s_1) > p(r|s_2)}$

$\hat{s} = s_i$ iff $p(r|s_i) \geq p(r|s_j) \quad \forall j \neq i$ $\quad i,j \rightarrow 1$ to M

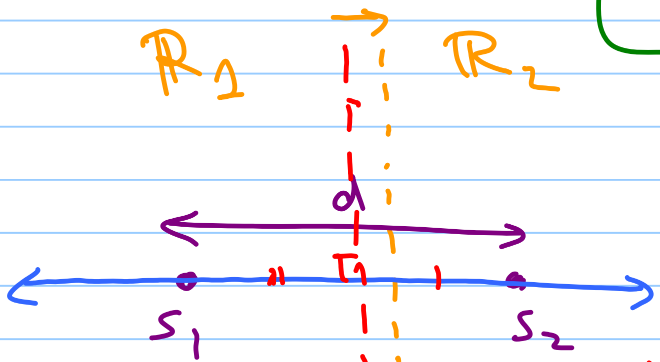
$$\cancel{\frac{1}{\sqrt{\pi N_0}}} e^{-\frac{(r-s_i)^2}{N_0}} \geq \cancel{\frac{1}{\sqrt{\pi N_0}}} e^{-\frac{(r-s_j)^2}{N_0}}$$

Voronoi

$$\hat{s} = s_i \text{ iff } \frac{-(r-s_i)^2}{\cancel{N_0}} \geq \frac{-(r-s_j)^2}{\cancel{N_0}}$$

$$\text{iff } \boxed{(r-s_i)^2 \leq (r-s_j)^2} \quad \forall j \neq i$$

Nearest Neighbour Rule based on Euclidean Distance ✓

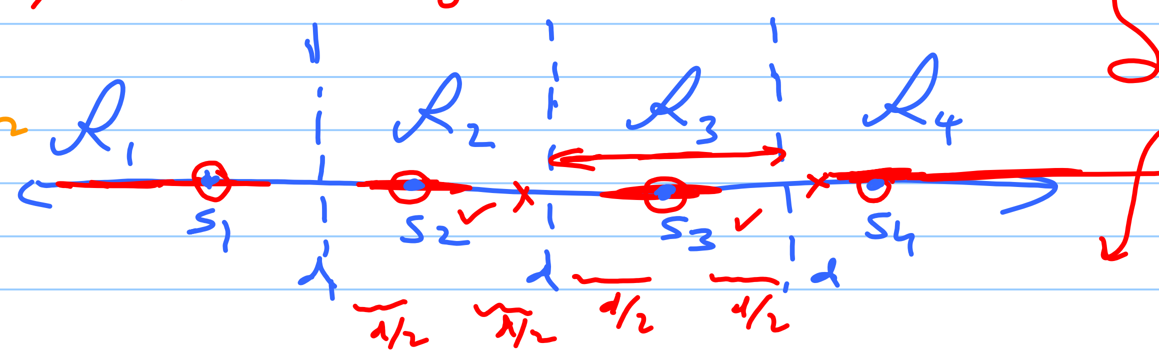


↳ decision boundary ✓

$p(s_1) > p(s_2)$,

$=$

$\mathbb{R}_1 > \mathbb{R}_2$



s_2, s_3 ✓

s_1, s_4 ✓