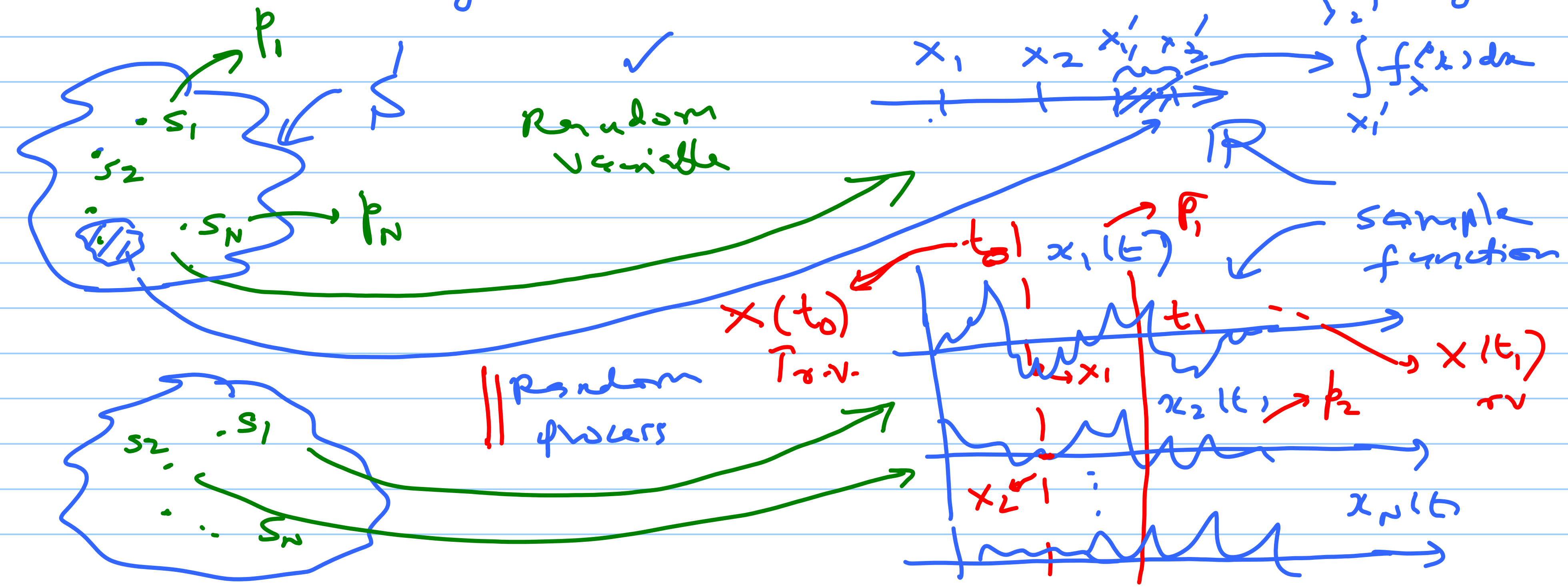
EE4140 **Class # 5** - August 26, 2020

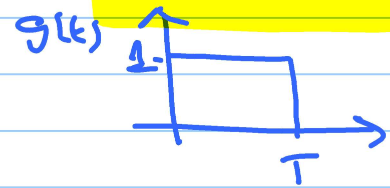
Note Title 3/27/2020

Random Binary Wave → Power Spectral Density.



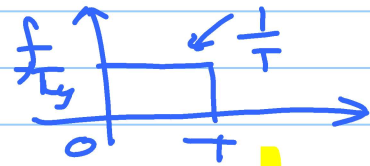
Random Binary Wave
(Random Pulse Train)

$$X(t) = \sum_{n=-\infty}^{+\infty} A_n \, g(t - nT - \tau_g)$$



$\in \{\pm A\} \Rightarrow P(A) = P(-A) = \frac{1}{2} \quad \forall n$

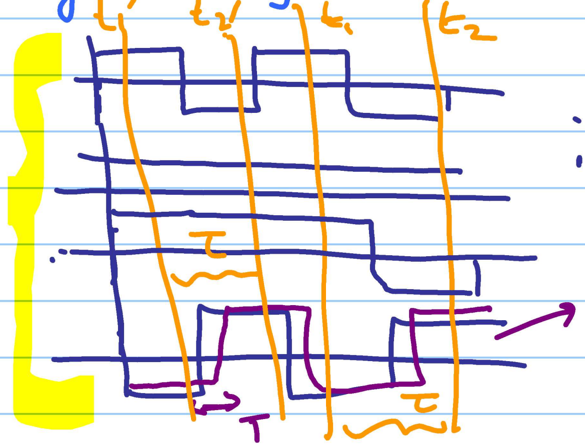
→ Each sample function experiences a random delay $0 \le \tau_g \le T$



$N = 4T$

2^4

2^{100}



$\tau_g = 0$ ⇒ cyclo-stationary

wide-sense

WSS process ✓

$m_X(t_0) = m = m_X(t_1)$

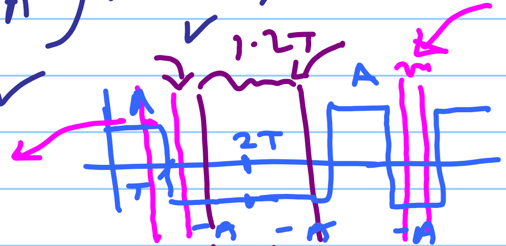
$R_X(t_1, t_2) = R_X(t_1 - t_2) = R_X(\tau)$

Find : (i) $E[X(t=t_i)] = m_x$

(ii) $E[X(t_1)X(t_2)] = E[X_1 X_2] = R_X(\overbrace{t_1, t_2}^{t_1 - t_2 = \tau})$ ✓

(i) $E[X(t_i)] = E[X(t_i) \mid +A]\,P[A] + E(X(t_i) \mid -A)\,P(-A)$

$= A \cdot \frac{1}{2} + -A \cdot \frac{1}{2} = 0$; ✓



(ii) Consider 2 cases to find $R_X(t_1, t_2)$

case A: $|t_1 - t_2| < T$ — x_1, x_2 correlated

case B: $|t_1 - t_2| \geq T$ — uncorrelated r.v.

Define $\Theta \triangleq \{ s : x(t)$ have a bit-boundary between t_1 & $t_2 \}$

$\bar{\Theta} \triangleq$ complementary event s.t. $P[\Theta] + P[\bar{\Theta}] = 1$

$$R_x(t_1, t_2) = E[x_1 x_2] = E[x_1 x_2 | \theta]\, P[\theta] + E[x_1 x_2 | \bar{\theta}]\, P[\bar{\theta}]$$

$$(*)\quad P[\theta] = \begin{cases} \dfrac{|t_1 - t_2|}{T}, & |t_1 - t_2| < T \\ 1, & |t_1 - t_2| \geq T \end{cases}$$

$$E[x_1 x_2 | \theta] = \sum \sum_{x_1, x_2} \underbrace{p_{x_1, x_2 | \theta}}_{\to\ p_{x_1} \cdot p_{x_2}}$$

$$\left.\begin{array}{ll} 25\% \to A\ A & 25\% - A\ A \\ 25\% \to A - A & 25\% - A - A \end{array}\right\} = \frac{1}{4}(A^2 + A^2 - A^2 - A^2) = 0$$

$$E[x_1 x_2 | \bar{\theta}] = \frac{1}{2}[A \cdot A + -A - A] = A^2;$$

$$P(\theta) = (1 - P(\bar{\theta}))$$

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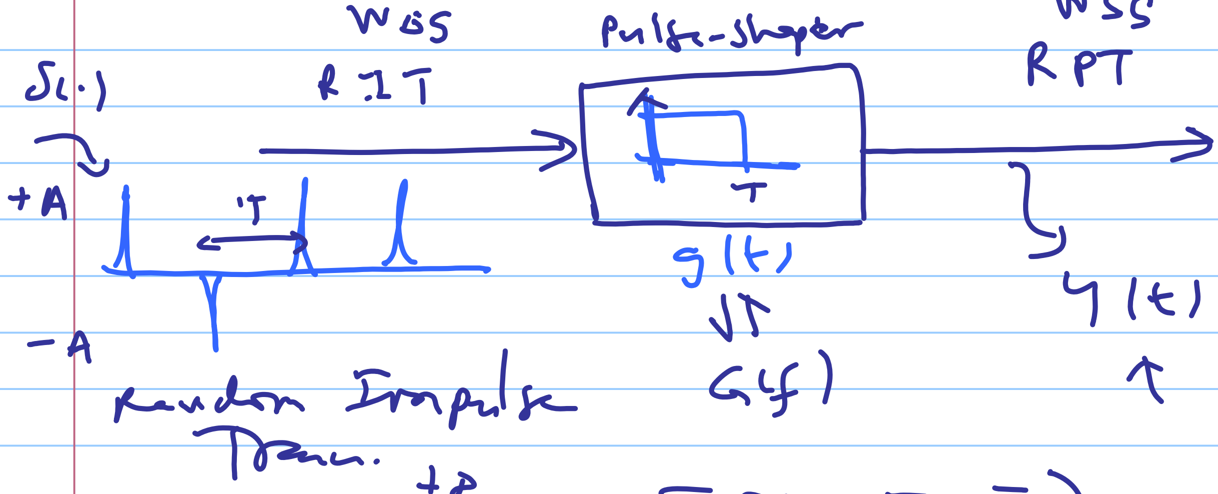
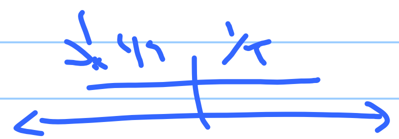
Random Binary Wave $\longrightarrow$ Random Pulse Train



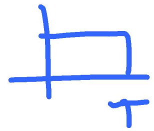
$$x(t) = \sum_{n=-\infty}^{+\infty} A_n \, \delta(t - nT - \tau_g)$$

t



$$\boxed{R_x(\tau) = A^2 \cdot \frac{\delta(\tau)}{T}}$$

$$R(T) \longrightarrow \boxed{g(t)} \longrightarrow Y(t)$$



$$R_Y(\tau) = E\left[\iint g(\tau_1)\, g(\tau_2)\, R_X(\tau - \tau_1 + \tau_2)\, d\tau_1\, d\tau_2\right]$$

$$R_Y(\tau) = \frac{A^2}{T} \int_{-\infty}^{+\infty} \underbrace{g(\tau_1)\, g(\tau_1 - \tau)\, d\tau}_{\text{acf of } g(t)}$$

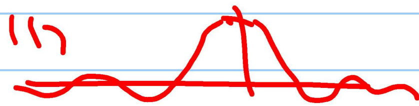
Linearly Modulating Binary Signal

$$S_Y(f) = \boxed{\frac{A^2}{T}\, |G(f)|^2}$$

$\neq f$

$T \to$ secs

$g(t)$



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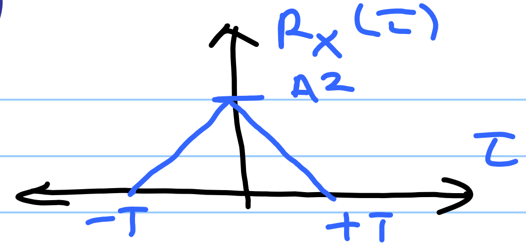
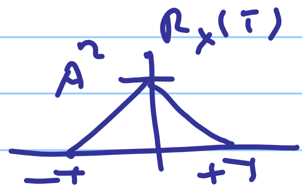
$$\therefore\ R_X(t_1, t_2) = A^2\, P(\bar{\Theta}) : A^2 (1 - P(\Theta))$$

$$R_X(t_1, t_2) = A^2\left(1 - \frac{|t_1 - t_2|}{T}\right)$$

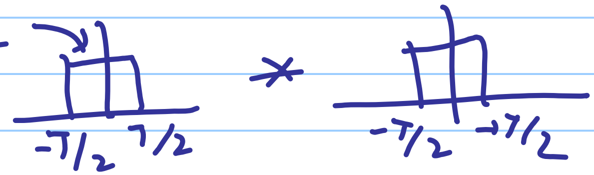


$$\Rightarrow\ R_X(\tau) = A^2\left(1 - \frac{|\tau|}{T}\right);$$

$$R_X(\tau) \rightleftharpoons S_X(f) \quad \text{PSD}$$

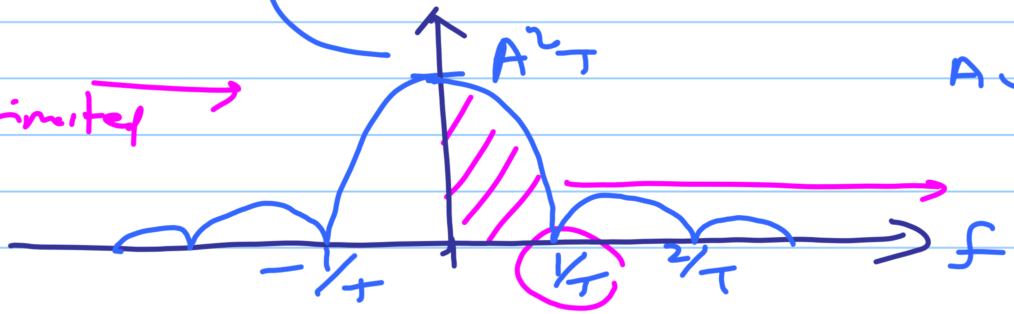


$\rightarrow$



$$A\sqrt{T} \cdot \operatorname{sinc}(\pi f T)$$

not bandlimited



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