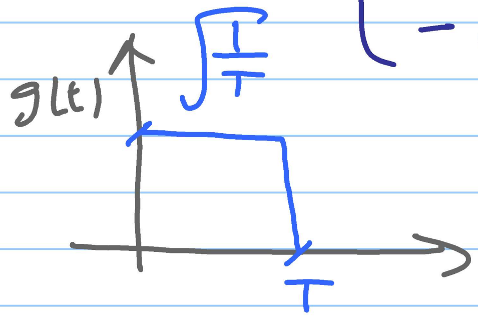
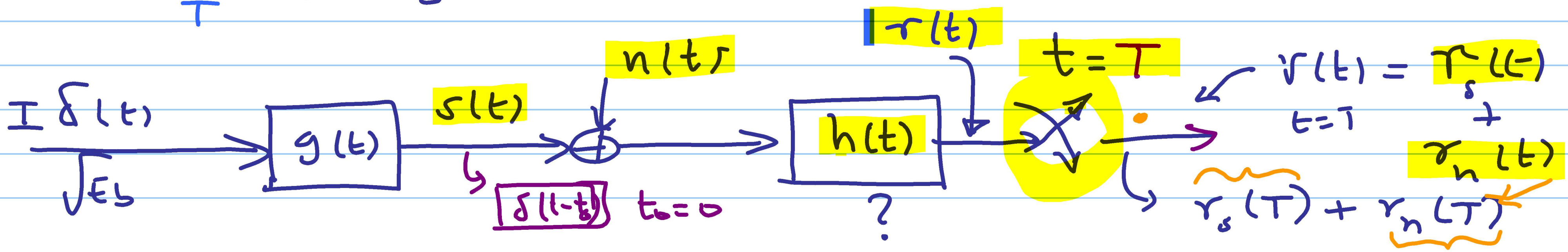
"Single Shot" Receiver

$$s(t) = \begin{cases} +\sqrt{E_b}\, g(t), & \text{if bit } I = \text{"1"} \\ -\sqrt{E_b}\, g(t), & \text{if bit } I = \text{"0"} \end{cases}$$



$$\int_0^T g^2(t)\,dt = 1$$



$n(t) \longrightarrow$ White Noise $\Rightarrow$ WSS

$$E\left[n^*(t)\, n(t-\tau)\right] = R_n(\tau) = \frac{N_0}{2}\,\delta(\tau)$$

? Auto corr. fn:

$$R_n(\tau) \rightleftharpoons S_n(f)$$

$\longrightarrow R_n(0) = \infty$

$$\int S_n(f)\cdot|H(f)|^2\, df$$

$$x(0) = \int_{-\infty}^{+\infty} X(f)\cdot df$$

$$SNR_o = \frac{r_s^2(T)}{E\left[r_N^2(T)\right]}$$

$$= \frac{\left(\int_0^T s(\tau)\, h(T-\tau)\, d\tau\right)^2}{E[\quad]}$$

$$r_N(T) = \int_0^T n(\tau)\, h(T-\tau)\, d\tau$$

$$= \int_0^T \int_0^T \underbrace{E\left[n(\tau)\, n(t)\right]}_{\frac{N_o}{2}\delta(t-\tau)} h(T-\tau)\, h(T-t)\, dt\, d\tau$$

$$= \frac{N_o}{2} \int_0^T \overbrace{\underbrace{h^2(T-t)\, dt}_{\text{Energy Rx filter}}}^{=1}$$

$$\therefore \; SNR_o = \left[\underbrace{\int_0^T s(\tau)\, h(T-\tau)\, d\tau}_{\tau}\right]^2 \Big/ \left(\quad \right)$$

$$\int_0 h(\tau)\, s(T-\tau)\, d\tau$$

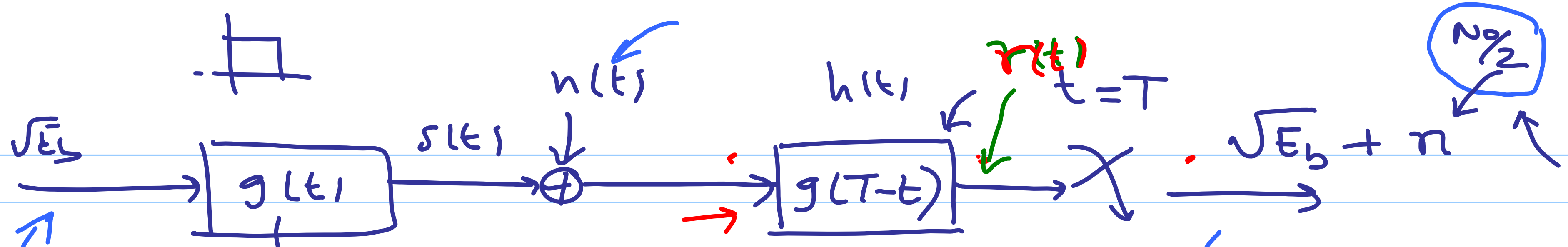
Recall Cauchy-Schwartz Inequality :

$$\left(\int_{-\infty}^{+\infty} x_1(t)\, x_2(t)\, dt \right)^2 \leq \int_{-\infty}^{+\infty} x_1^2(t)\, dt \int_{-\infty}^{+\infty} x_2^2(t)\, dt$$

Aside : $\bar{x}_1$ & $\bar{x}_2$ $[\;]_{N\times 1}$

$$\left(\bar{x}_1^T \bar{x}_2 \right)^2 \leq \underbrace{\left(\bar{x}_1^T \bar{x}_1 \right)}_{\|\bar{x}_1\|_2^2} \underbrace{\left(\bar{x}_2^T \bar{x}_2 \right)}_{\|\bar{x}_2\|_2^2}$$

Equality iff $\boxed{x_2(t) = C\, x_1(t)}$



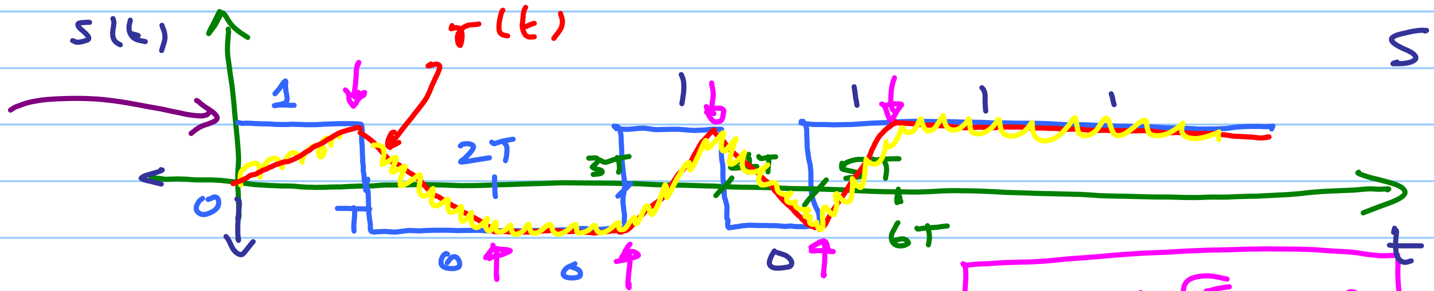
$\sqrt{E_s}$

$\pm\sqrt{E_b} \pm j\sqrt{E_b}$

$E_s =$ $\boxed{\dfrac{E_s}{N_0}} \rightarrow \boxed{\dfrac{2E_b}{N_0}}$

$\therefore \; SNR_o = \dfrac{(\sqrt{E_b})^2}{N_0/2} = \boxed{\dfrac{2E_b}{N_0}}$

$$\frac{\text{Signal Power}}{\text{Noise Variance}}$$



$$s(t) = \sum_{k=0}^{\infty} I_k \, g(t-kT)$$

$$I_k \in \{+\sqrt{E_b}, -\sqrt{E_b}\}$$

$$\boxed{r = \pm\sqrt{E_b} + n}$$