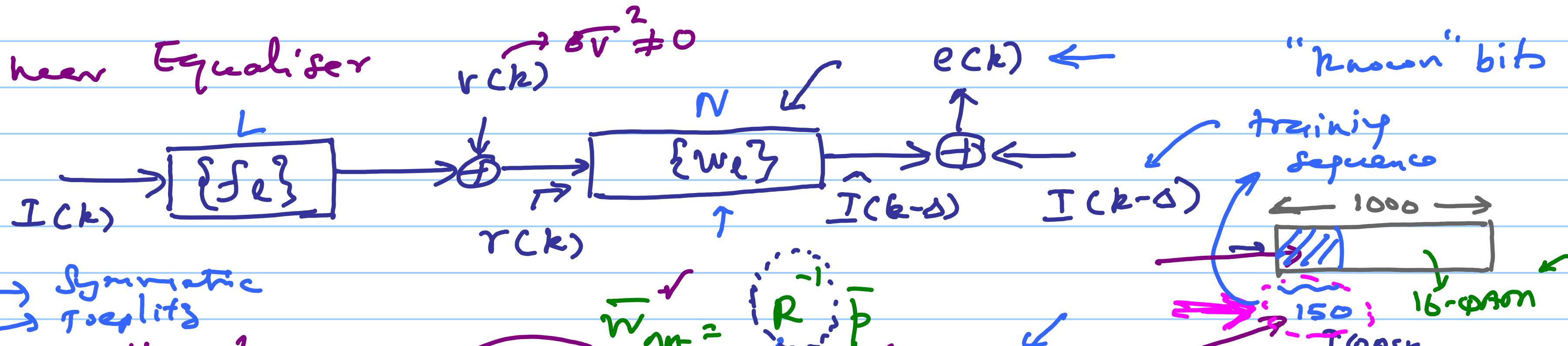
EE4140 -- Lecture #26 Nov. 20, 2020

Note Title 09-11-2020

Linear Equaliser



Symmetric
Toeplitz
$R \to$ full-rank

$\bar{w}_{opt} = R^{-1} \bar{p}$

$R = E[\bar{r}\,\bar{r}^T]$

$O(N^3)$

$\tilde{\bar{p}} = \frac{1}{P} \sum_{i=1}^{P} I(i)\, \bar{r}_i$

$\frac{1}{P} \sum_{i=1}^{P} \bar{r}_i\, \bar{r}_i^T = \tilde{R}$

$[r(k) \quad r(k-N+1)]$

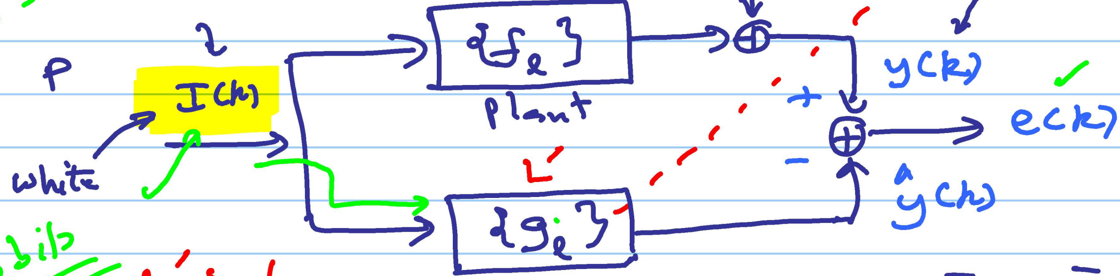
Toeplitzation

$$R = \underbrace{\begin{bmatrix} \cdot + \sigma_v^2 & & \\ & \ddots & \\ & & \cdot + \sigma_v^2 \end{bmatrix}}_{N \times N}$$

$\{\lambda_i\} \leftrightarrow \{f_\ell\}$

eigen-value spread $\frac{\lambda_{max}}{\lambda_{min}}$

System ID Problem ← virtual for MLSE - VA ←



Solution: $L' \geq L$

→ Model order estimation

Assume $L' = L$;

Exercise: using the L-MMSE formulation show that

$\bar{g} = R^{-1} \bar{p}$ → $\tilde{R}, \tilde{p}$

$\bar{g} \to \bar{f}$

$L \times L$

R

$R_1 \to$ diagonal matrix ← 1

$\lambda_{max} = \lambda_{min}$

Equaliser

→ Adaptive Estimation of → $\bar{W}$, $\bar{f}\ (\bar{g})$ ‖ gradient descent

N=2

$\bar{w}(k) = \bar{w}(k-1) + 2\mu e(k)\, \bar{r}(k)$

$N \times 1$ $\qquad N \times 1$

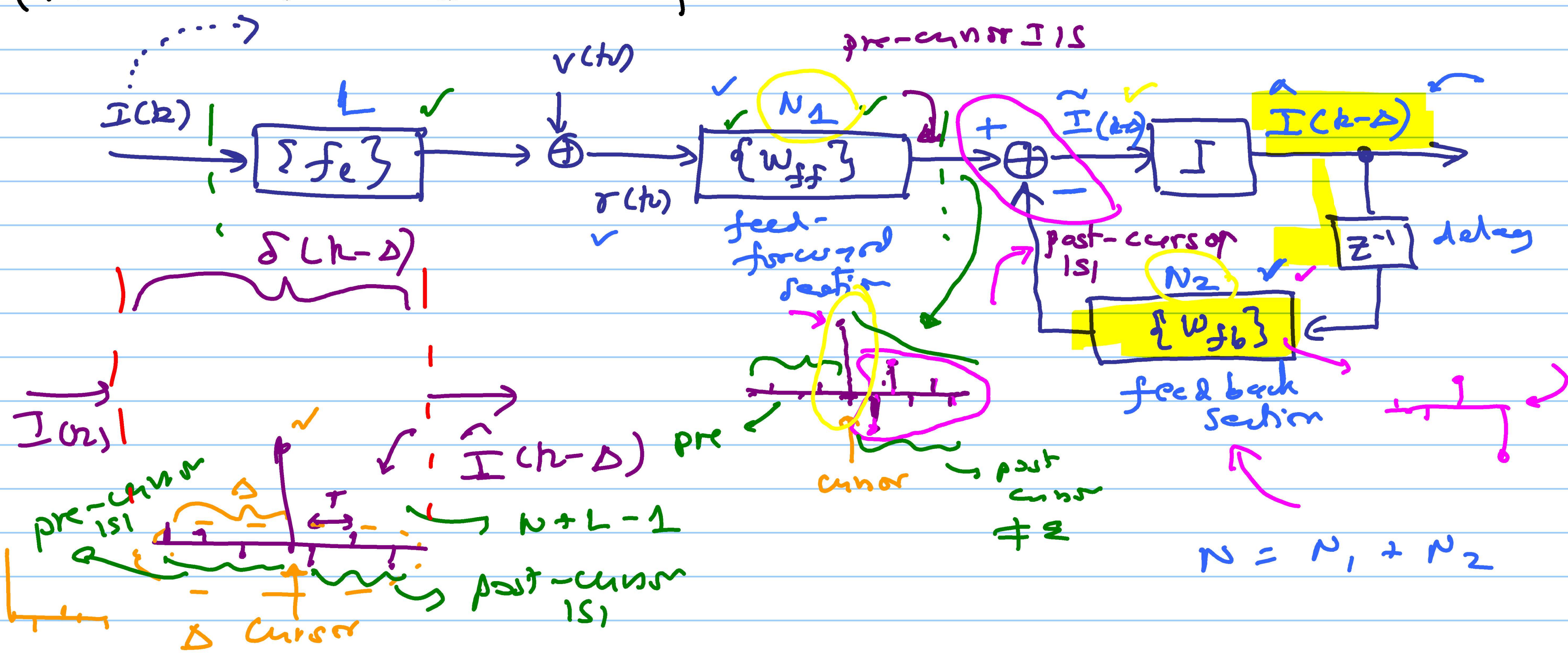
$\bar{W}_{opt}$

gain const.

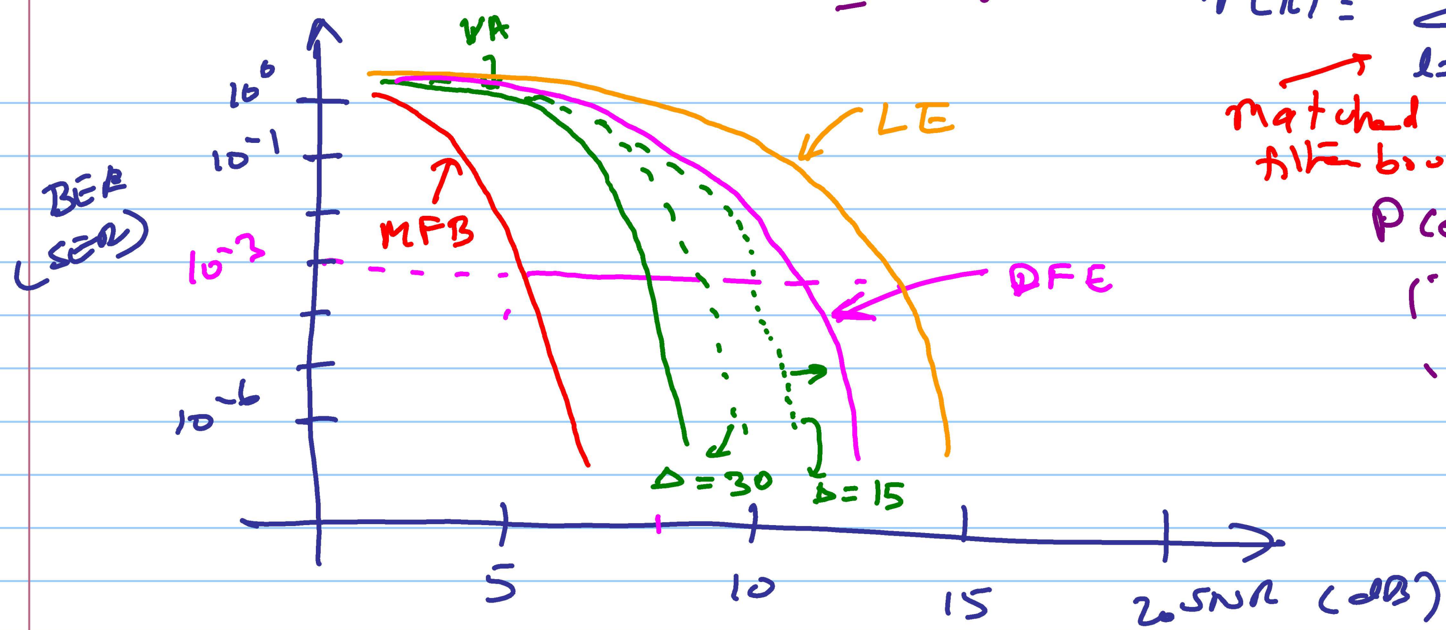
$I(k) - \hat{I}(k)$

e

(*) Decision Feedback Equalisation ← Non-linear structure



$$\underline{r} = \underline{I} + \underline{v}$$



$$r(k) = \sum_{l=0}^{L-1} f_l\, I(k-l) + v(k)$$

matched filter bound

$$P(e) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{(\alpha) E_s}{N_0}}\right)$$

$$\sum_{l=0}^{L-1} f_l^2 = \alpha$$

$$R = E\left[\bar{r}\,\bar{r}^{T}\right] \; ; \quad \bar{r}$$

DFE

$$\underset{\bar{w}}{\min}\, E\left[e^2(n)\right]$$

$$\underset{N\times 1}{\bar{w}} = \begin{bmatrix} \bar{w}_{ff} \\ \bar{w}_{fb} \end{bmatrix} \begin{matrix} \rightarrow N_1 \times 1 \\ \rightarrow N_2 \times 1 \end{matrix}$$

$N_1 + N_2 = N$

$$\bar{r} = \begin{bmatrix} r(k) \\ r(k-1) \\ \vdots \\ r(k-N_1+1) \\ \hat{I}(k-\Delta-1) \\ \hat{I}(k-\Delta-2) \\ \vdots \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} a\\a\\a\\a \end{matrix}}\right\} N_1 \\ \left.\vphantom{\begin{matrix} a\\a\\a \end{matrix}}\right\} N_2 \end{matrix}$$

$$\underset{N\times 1}{\bar{p}} = E\left[I(k-\Delta)\cdot \bar{r}\right]$$

$$\boxed{\bar{w} = R^{-1}\,\bar{p}}$$

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