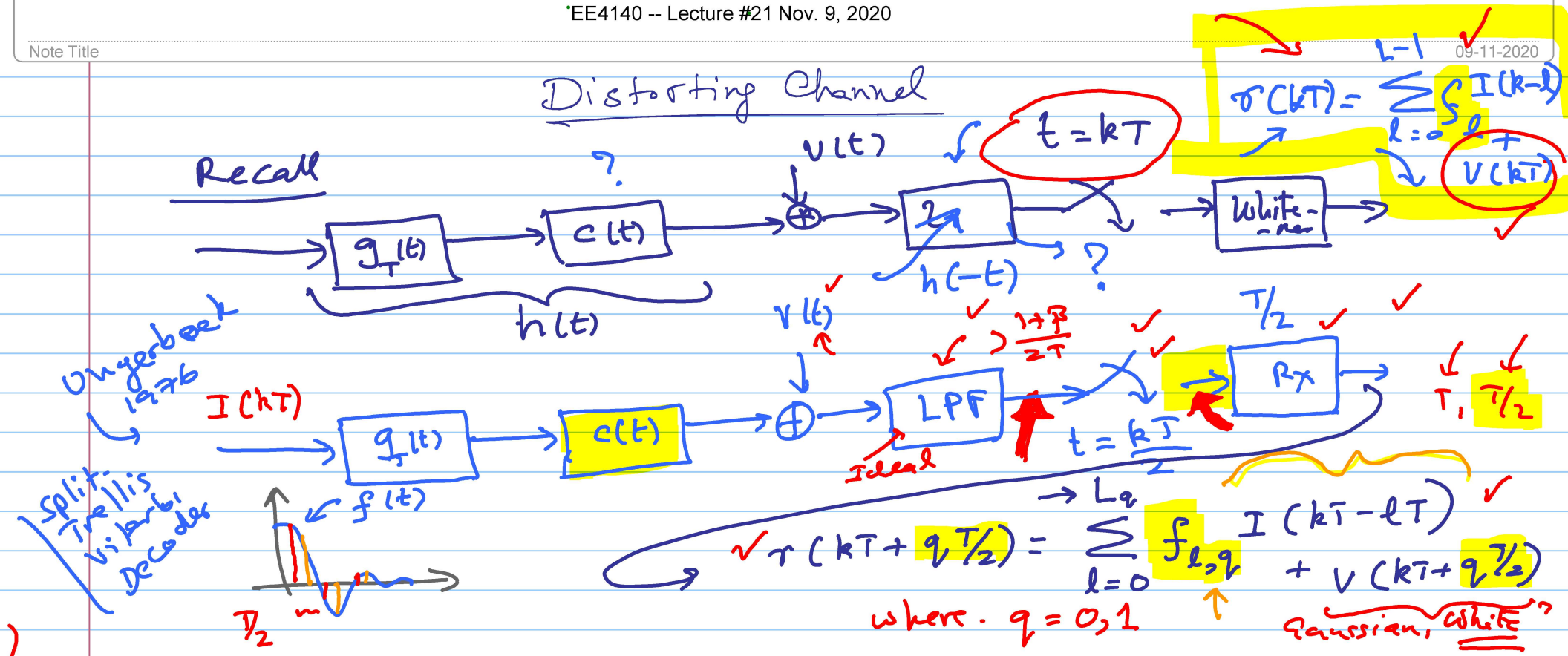
EE4140 -- Lecture #21 Nov. 9, 2020

Note Title 09-11-2020



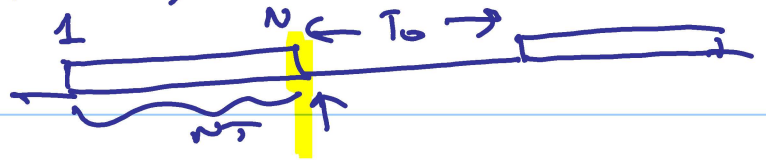
$I(1), I(2), \ldots, I(N)$

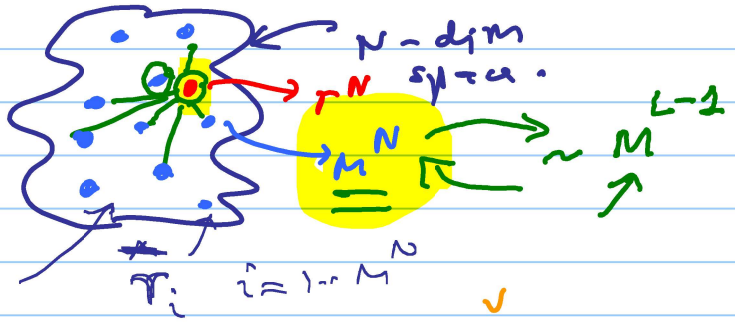
$I^{N,1}$ I^{N}_{L-1} $I^{N,N-L+1}$

$$r(k) = \sum_{l=0} f_l \, I(k-l) + v(k)$$

r^N M^N $m=4$

NT intervals of time.





MAP Rule

Pick $\hat{I}^N = I^N_i$ iff $p(I^N_i \mid r^N) \geq p(I^N_j \mid r^N)$,

$\forall j \neq i$, $j = 1, 2, 3, \ldots, M^N$

MAP sequence

Sequence Estimation

$i = 1, \ldots, N$

$$\rightarrow \frac{p(r^N \mid I_i^N)\, p(I_i^N)}{p(r^N)} \geq \left(\frac{p(r^N \mid I_j^N)\, p(I_j^N)}{p(r^N)} \right)$$

Assuming $I(N)$ is coming from a uniform Alphabet $p(I_i) = \frac{1}{M} \; \forall i$

$$p(I_i^N) = \frac{1}{M^N}$$

Maximum likelihood Rule : (ML)

$$\rightarrow p(r^N \mid I_i^N) \geq p(r^N \mid I_j^N) \leftarrow \text{ML}$$

$$r(k) = \sum_{l=0}^{L-1} f_l\, I(k-l) + V(k)$$

$I(k-l)$: Gaussian

$$\begin{bmatrix} V(1) \\ V(2) \\ \vdots \\ V(N) \end{bmatrix}$$

Jointly Gaussian

$$\bar{r} = [\mathbf{I}]\, \bar{f}_l + \bar{V}$$

$\bar{V}$

MAP Rule

$$\frac{1}{\sqrt{2\pi \det(R_v)}} e^{-\frac{1}{2}(\bar{r}-\bar{r}_i)^T R_v^{-1} (\bar{r}-\bar{r}_i)} \gtrless \frac{1}{\sqrt{2\pi \det(R_v)}} e^{-\frac{1}{2}(\bar{r}-\bar{r}_j)^T R_v^{-1} (\bar{r}-\bar{r}_j)}$$

$\bar{v}$ = white, $R_v = \sigma_v^2 I_{N\times N}$

$\forall j \neq i, 1 \cdots M^N$

$$\prod_{k=1}^{N} e^{-\frac{1}{2}\frac{(r(k)-r_i(k))^2}{\sigma_v^2}} \gtrless \prod_{k=1}^{N} e^{-\frac{1}{2}\frac{(r(k)-r_j(k))^2}{\sigma_v^2}}$$

$\forall j \neq i, 1 \cdots M^N$

Monotic $\Rightarrow \log(\ln)$

$$\sum_{k=1}^{N} (r(k)-r_i(k))^2 \lessgtr \sum_{k=1}^{N} (r(k)-r_j(k))^2$$

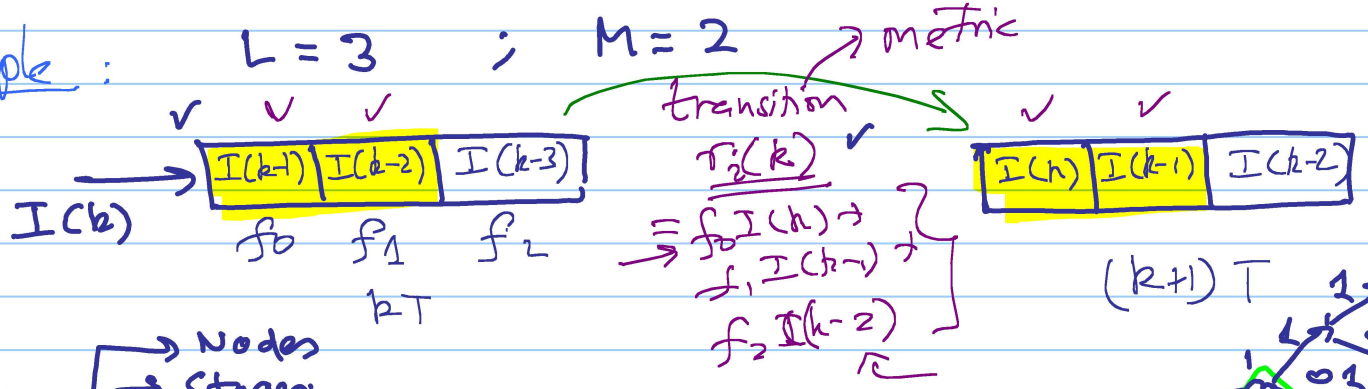
Euclidean Distance.

$\forall j \neq i, 1 \cdots M^N$

Viterbi Algorithm for MLSE / MAPSE

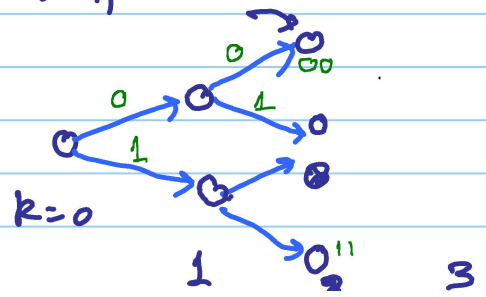
SE | Fano Alg.
Stack Alg.

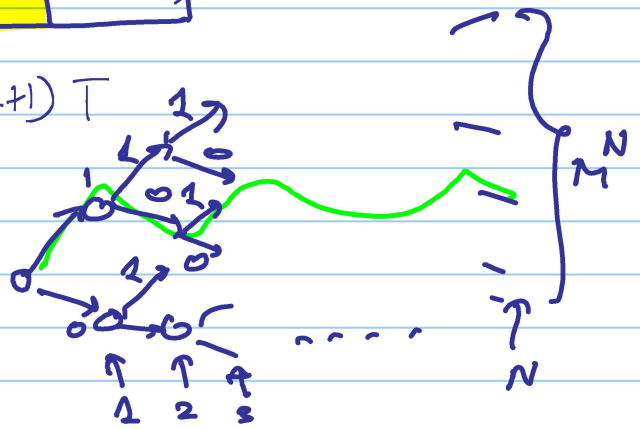
Example: $L = 3$; $M = 2$

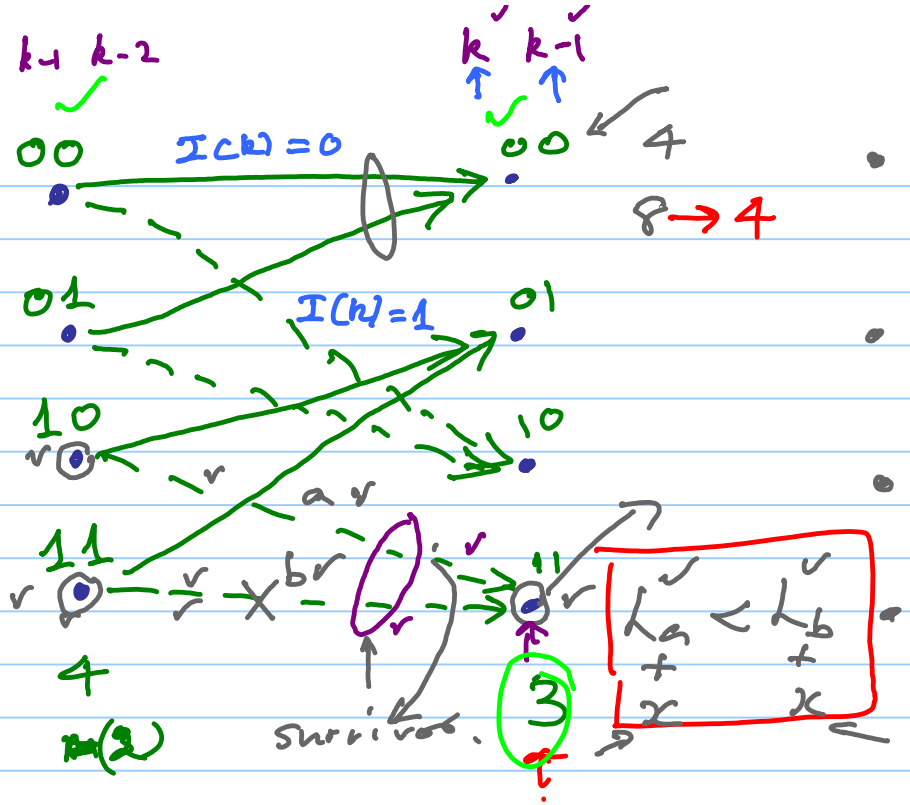


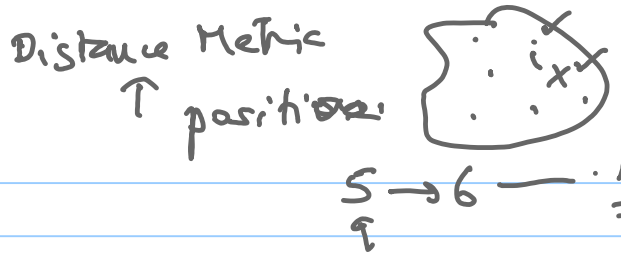
Trellis → Nodes, Stages, Transition

# of Nodes in the trellis $= M^{L-1}$









$|\bar{x}_1| < |\bar{x}_2|$

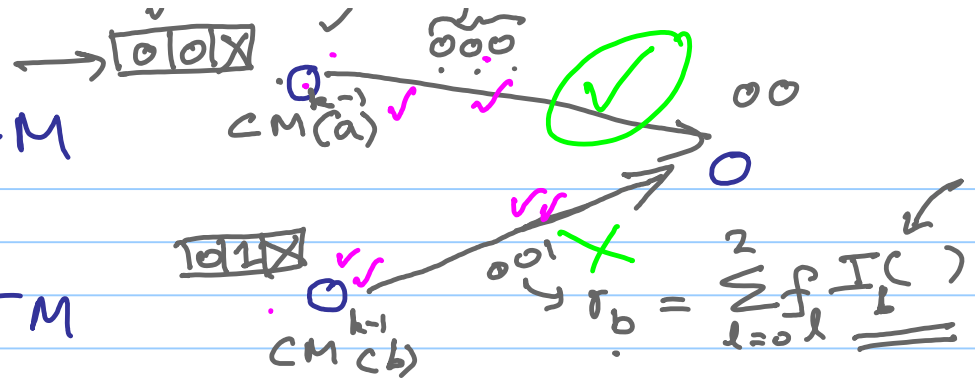
$|\bar{x}_1 + \bar{x}_3| < |\bar{x}_2 + \bar{x}_3|$

$TM_a = (r - r_a)^2$

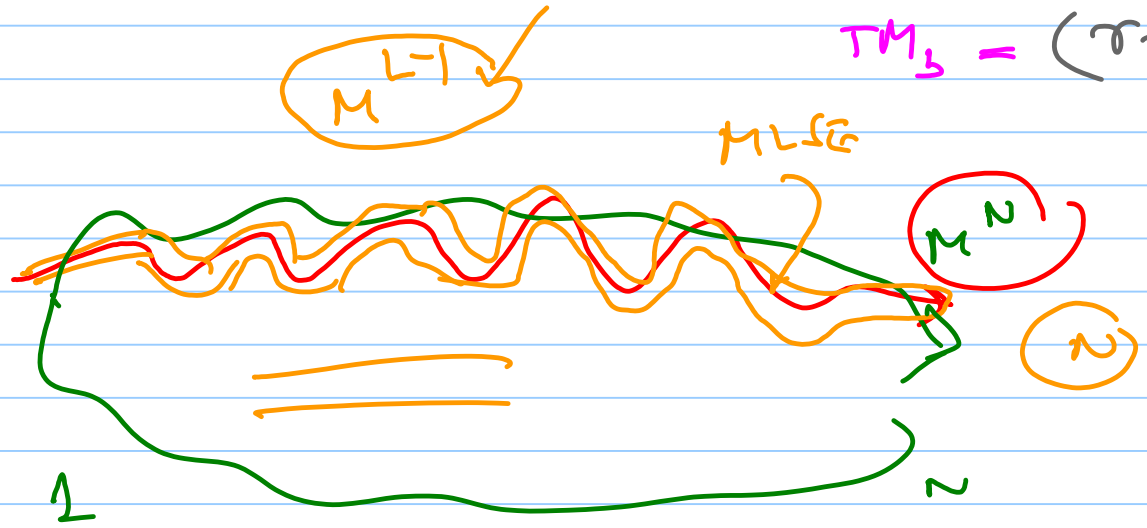
k^{th} r_a $k+i^{th}$

Cumulative Metric CM

Transition Metric TM



$TM_3 = (r - r_b)^2$



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