$P(e)$ for a Rayleigh fading Channel

$h(t) = \delta(t - t_d)$

$h(t) = c(t)\,\delta(t)$

$$r(t) = c(t)\, s_i(t) + n(t) \qquad i = 1 \cdots M$$

$n(t)$ ↳ AWGN

Circular Gaussian Random Process

$$c(t) = c_I(t) + j\, c_Q(t)$$

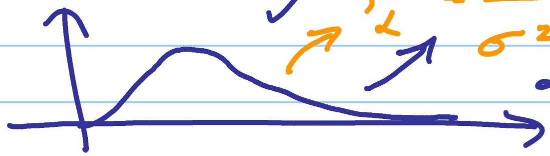
$\mathcal{N}(0, \sigma^2)$

$c_I \perp^r c_Q$

Rayleigh Envelope

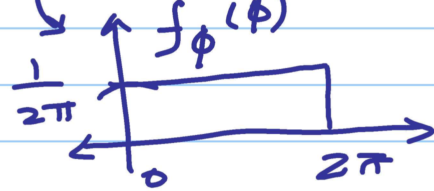
$$\alpha(t) = \sqrt{c_I^2(t) + c_Q^2(t)} \; ; \quad \phi(t) = \tan^{-1} \frac{c_Q(t)}{c_I(t)}$$

$f_\alpha(\alpha)$



$$f_\alpha(\alpha) = \frac{\alpha}{\sigma^2} e^{-\alpha^2/2\sigma^2}, \quad \alpha \geq 0$$

$f_\phi(\phi)$

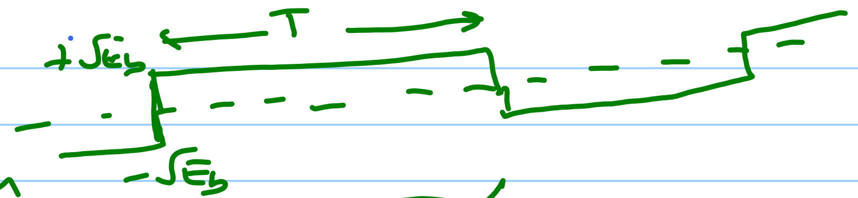


$$\boxed{c(t) = \alpha(t)\, e^{j\phi(t)}}$$

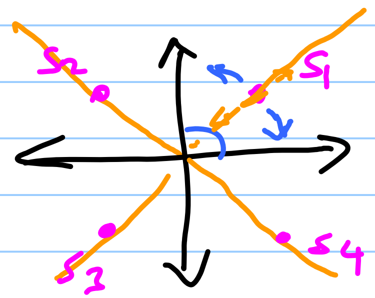
Assuming a slowly fading frequency non-selective model

$$m(t) = \sum_k I(k)\, g(t-kT)$$

symbol. duration



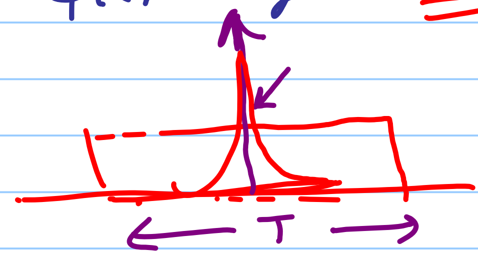
How much does $\alpha(t)$ & $\phi(t)$ change in T secs?



$\sqrt{E_b}$

$$\boxed{C = \alpha e^{j\phi}}$$

NT



BPSK / Tx $\rightarrow$ $s_m(t) = \sqrt{\frac{2E_b}{T}} \cos(2\pi f_c t + m\pi)$, $m = 0, 1$

Rx $\rightarrow$ $r(t) = \alpha \sqrt{\frac{2E_b}{T}} \cos(2\pi f_c t + m\pi + \phi) + n(t)$

$kT \leq t \leq (k+1)T$

NT

Assuming that the R_x can generate (locally) $\sqrt{\frac{2}{T}}\cos(2\pi f_c' t + \phi')$

$\Rightarrow$ ideal freq. sync + phase-sync. $\rightarrow$ PLL

$\cos(2\pi \Delta f t)$

T_x: clk $\rightarrow f_c$, clk $\rightarrow T$

R_x: clk $\rightarrow f_c'$, clk $\rightarrow T'$

$r(t) \rightarrow \otimes \rightarrow$ LPF $\rightarrow$ ADC (kT) $\rightarrow y$

$\sqrt{\frac{2}{T}}\cos(2\pi f_c t + \phi)$

$n \rightarrow \frac{N_0}{2}$

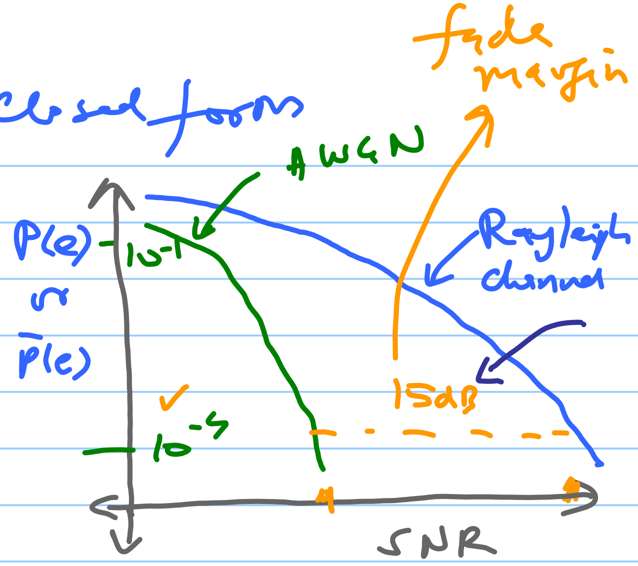
$$\boxed{y = \underbrace{\alpha\sqrt{E_b}}\cos m\pi + n, \quad m = 0, 1}$$

$\rightarrow y = \sqrt{E_b}\cos m\pi + n \rightarrow Q\left(\sqrt{E_b/N_0/2}\right)$

$$P_e(\alpha) = Q\left(\sqrt{\frac{\alpha^2 E_b}{N_0/2}}\right)$$

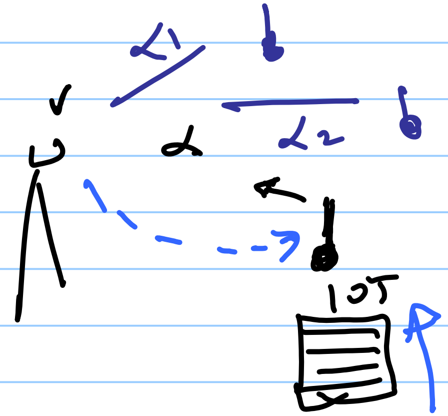
$$\bar{P}(e) = \int_0^\infty P_e(\alpha) f_\alpha(\alpha)\, d\alpha$$

closed form



$$\bar{P}(e) = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\rho}_b}{1+\bar{\rho}_b}}\right]$$

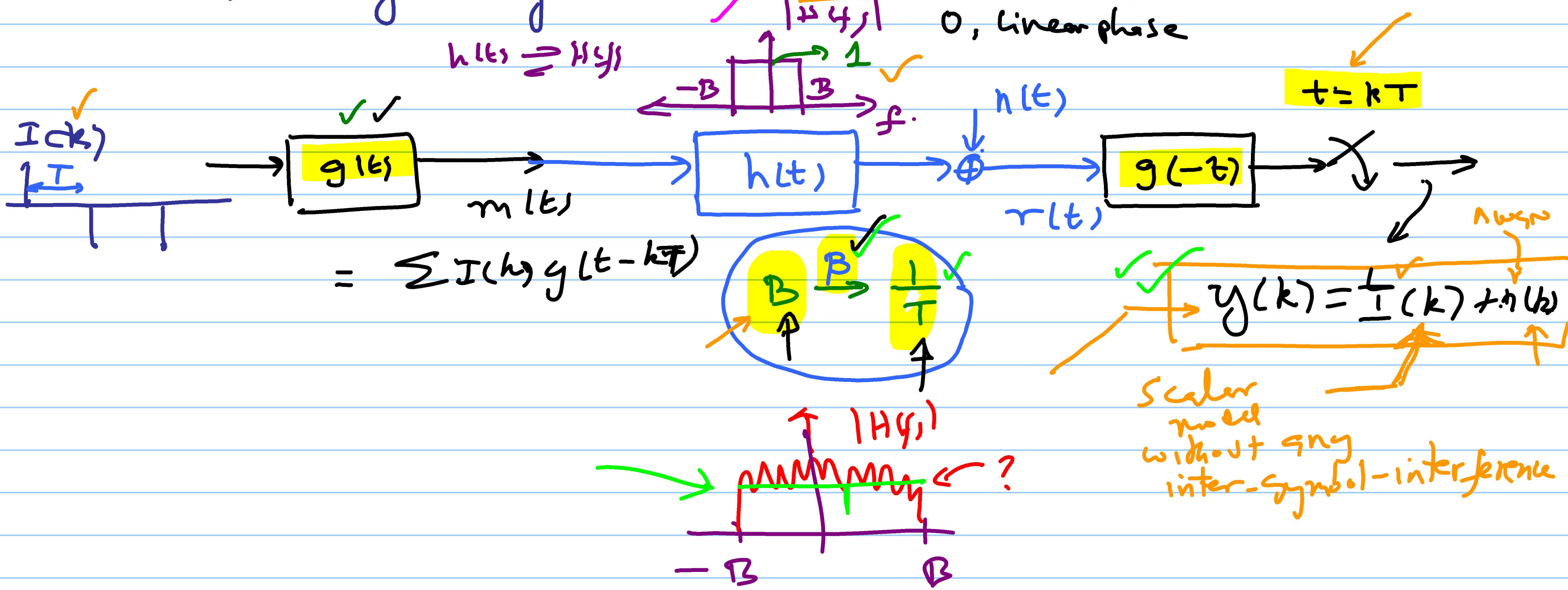
where $\bar{\rho}_b = \frac{E_b}{N_0} \underbrace{E(\alpha^2)}_{2\sigma^2}$



Rayleigh

$$r = \alpha s_i + n \qquad i \in 1, 2 \cdots M$$

2. Signalling Thro' Ideal Band-Limited Channels

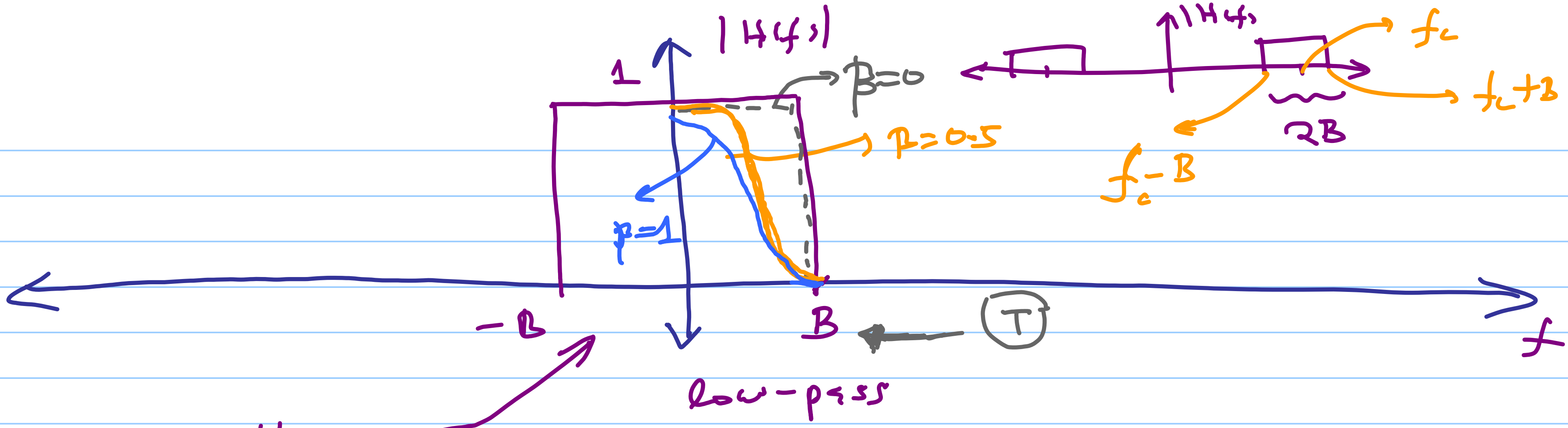


$$m(t) = \sum I(k)\, g(t-kT)$$

$$B \rightarrow \frac{1}{T}$$

$$y(k) = I(k) + n(k)$$

Scaler model without any inter-symbol-interference



$g(t)$

$\hookrightarrow$ sinc(·) $\rightarrow$ $\beta = 0$ $\rightarrow$ $\frac{1}{2T} = B$

$\hookrightarrow$ RC $\quad 0 < \beta \leq 1 \rightarrow \frac{(1+\beta)}{2T}$ ||