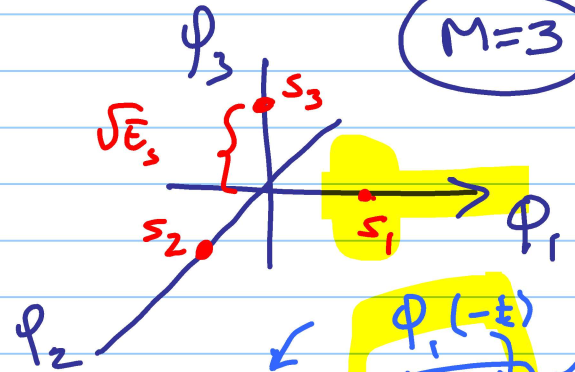
Example: orthogonal signals $P(e)$

$N = M$ $\qquad s_i(t) = \sqrt{E_s}\,\phi_i(t)$, $i = 1 \dots M$ $\qquad 3 \quad 2^k \quad 4$

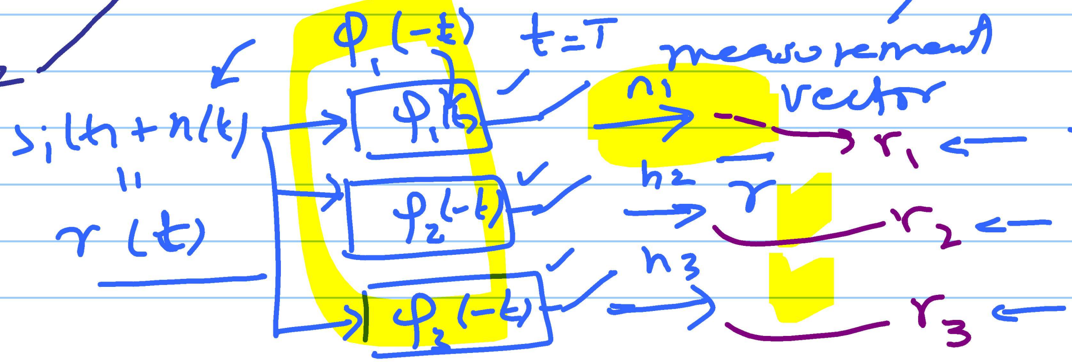
$M=3$

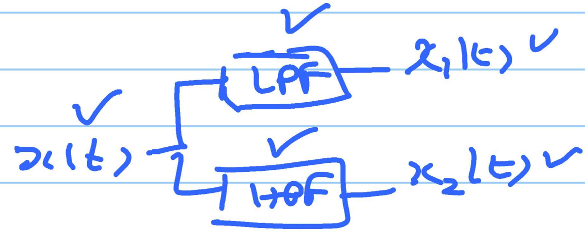


$\bar{r} = \bar{s}_i + \bar{n}$

$s_i = s_1$

$$\bar{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} \sqrt{E_s} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

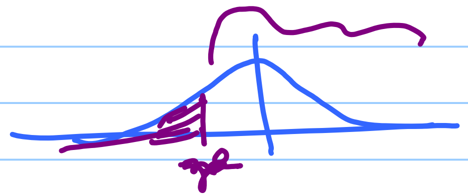




$$P(c) \rightarrow P\left[\bar{r} \in Z_1 \,\middle|\, \bar{s}_1 \text{ sent}\right] = \int_{-\infty}^{+\infty} P\left[\bar{r} \in Z_i \,\middle|\, \bar{s}_1, r_1 = \vartheta\right] P[r_1 = \vartheta]\, d\vartheta$$

Here, $P\left[\bar{r} \in Z_1 \,\middle|\, \bar{s}_1, r_1 = \vartheta\right] = P\left[r_2 < \vartheta, r_3 < \vartheta\right]$

$$= \underbrace{P\left[n_2 < \vartheta, n_3 < \vartheta\right]}$$



$$= \left[1 - \int_{\vartheta}^{\infty} f_n(\alpha)\, d\alpha\right]^2$$

$$P[r_1 = \vartheta] = f_N\left(\vartheta - \sqrt{E_s}\right) \qquad f_N(\alpha) = \frac{1}{\sqrt{\pi N_0}} e^{-\alpha^2/N_0}$$

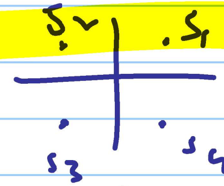
Finally

$$P(C) = P\left[\bar{r} \in Z_1 \mid \bar{s}_1\right] = \int_{-\infty}^{+\infty} f_N\left(r - \sqrt{E_s}\right)\left(1 - \int_{-\infty}^{r} f_N(\alpha)\, d\alpha\right)^{\cancel{2}\ M-1} dr$$

M

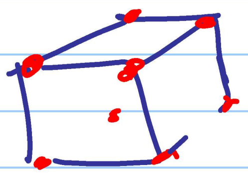
$P(e) = 1 - P(c)$

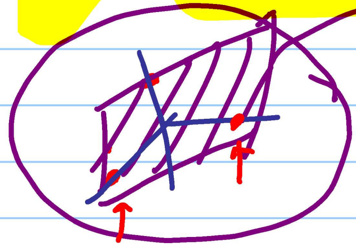
$M = 4$



$E[s_i] = 0$

zero DC

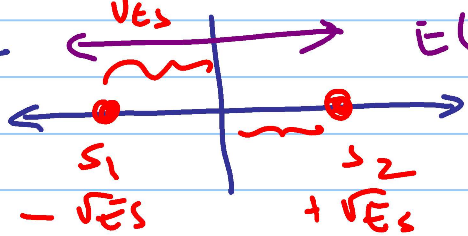




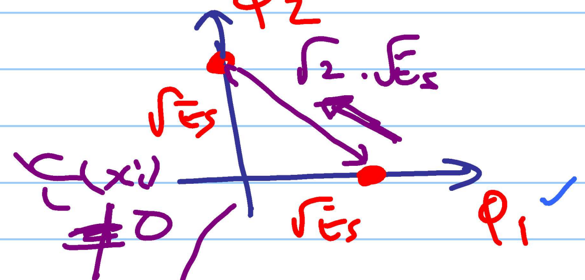
$DC \neq 0$

$E_a \longrightarrow$ high.

$M = 2$

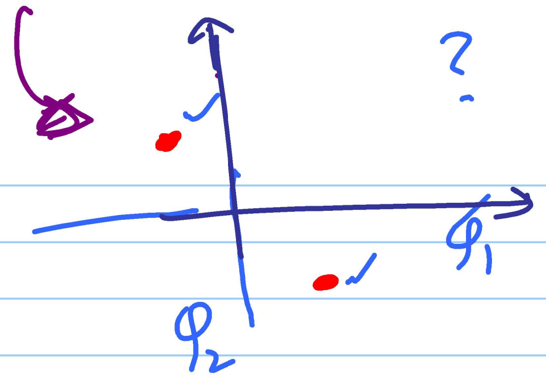


$E(s_i) = 0$



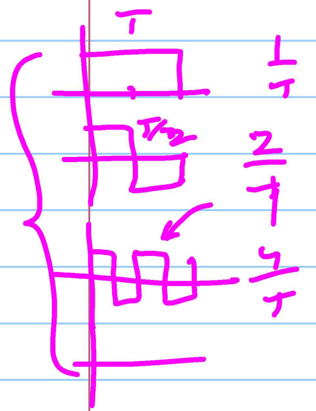
simplex signalset

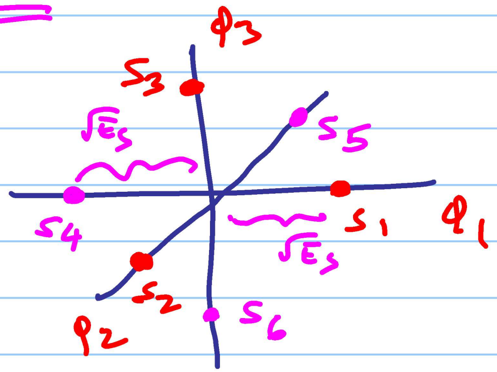
$$\begin{bmatrix}\sqrt{E_s}\\0\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\\sqrt{E_s}\\0\\0\end{bmatrix}, \quad \begin{bmatrix}0\\0\\0\\\sqrt{E_s}\end{bmatrix}$$



Exercise : Bi-orthogonal Signal Set $\quad M = 2N$

Find the $P(c) \longrightarrow P(e)$





$E_s = \cancel{k} E_b$

$M = 2^k$

As $M \uparrow$, what happens to $P(e)$?

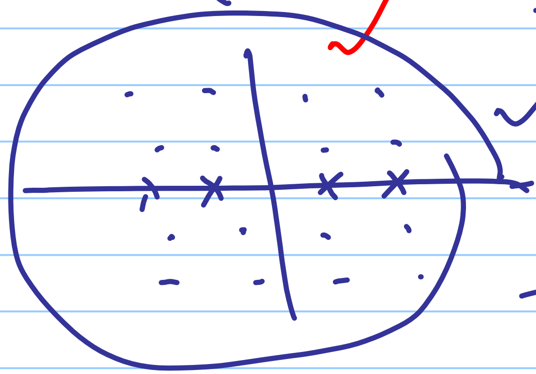
$P(c)$?

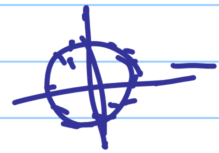
(for same E_b)

$N = 2$

$p(e) \uparrow$

$M \uparrow$



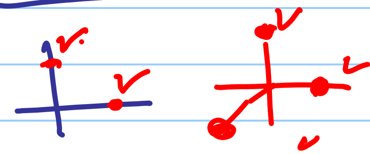


$\leftrightarrow d_{min}$

$N, M \uparrow$

$\uparrow M$, $N \uparrow$

$\sqrt{E_s}$ same



$P(e) \uparrow$