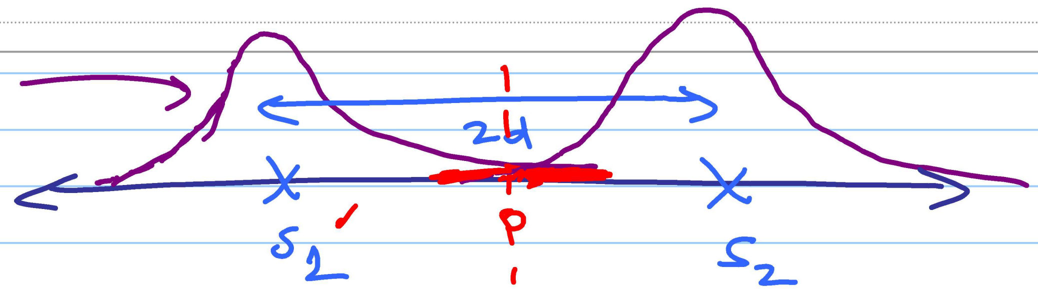
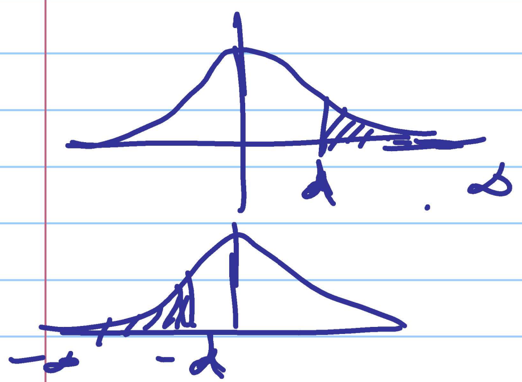
EE4140 **Class # 12** - October 02, 2020



$p(s_1) = p(s_2) = \frac{1}{2}$

$r = s_i + n$

$\mathcal{N}(0, \frac{N_0}{2})$

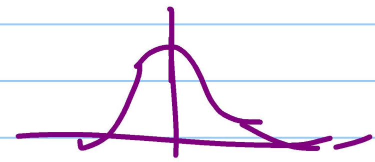


$$q = \frac{1}{2}\,\mathrm{erfc}\left(\frac{d}{\sqrt{N_0}}\right)$$

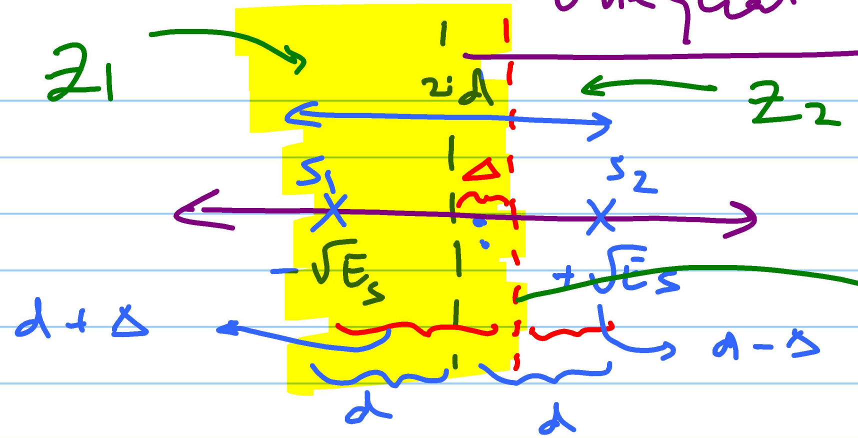
$$= Q\left(d/\sqrt{N_0/2}\right)$$

$n_1 + j n_2$

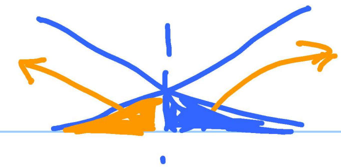
$n_I + j n_Q$



Unequal Priors



$p_1 > p_2$



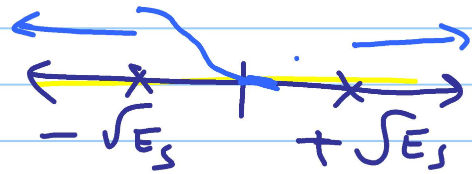
At the $\underline{\text{decision boundary}}$, the 2 MAP metrics must be equal.

$r = s_i + n$

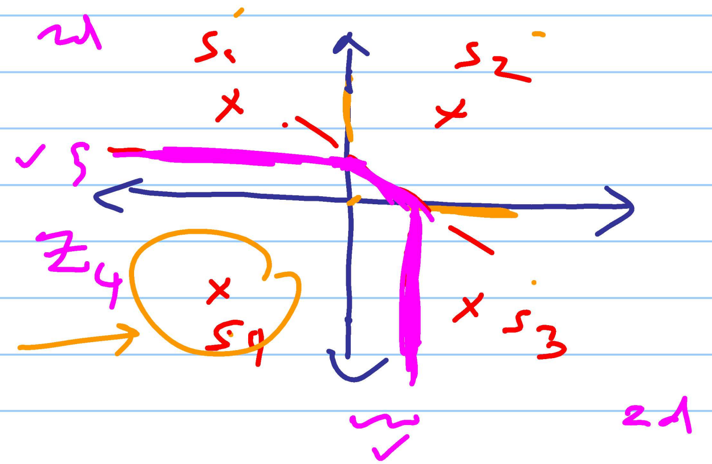
$$\underbrace{p(r \in Z_1 | s_1)} \cdot p(s_1) = p(r \in Z_2 | s_2)\, p(s_2)$$

$$\frac{1}{\sqrt{\pi N_0}}\, e^{-\frac{(d+\Delta)^2}{N_0}}\, p(s_1) = \frac{1}{\sqrt{\pi N_0}}\, e^{-\frac{(d-\Delta)^2}{N_0}}\, p(s_2)$$

$$\Delta = \frac{N_0/2}{2d} \ln\left(\frac{p_1}{p_2}\right) \Rightarrow \quad p_1 > p_2$$



$\sqrt{2} \cdot 2d$

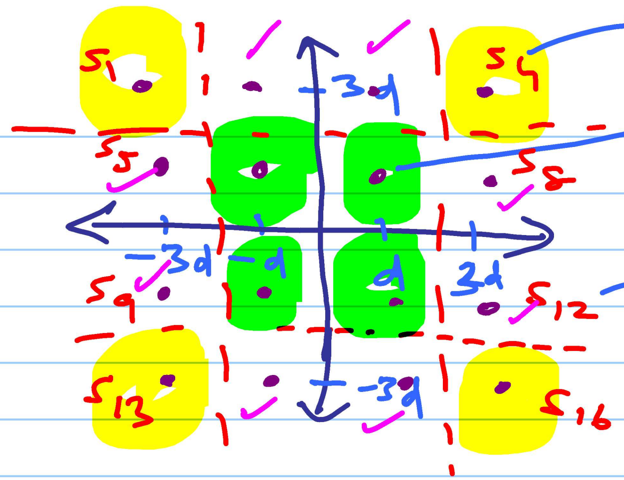


$p(s_4) = \frac{\cancel{7}}{10}$;

$p(s_1) = p(s_2) = p(s_3) = \frac{1}{10}$

Example # 4 $\quad P(e)$

16-QAM contellation



$\to (1-q)^2$

$\to (1-2q)^2$

$\to (1-q)(1-2q)$

$r = s_i + n$

$\mathcal{N}\left(0; 0; \frac{N_0}{2}; \frac{N_0}{2}\right)$

(1) $s_1, s_4, s_{13}, s_{16} \longrightarrow P(C) \longrightarrow P_{c1}$

(2) $s_6, s_7, s_{10}, s_{11} \longrightarrow P_{c2}$

(3) $\{ \quad \} \longrightarrow P_{c3}$

$$\boxed{P(c) = \frac{1}{4}P_{c1} + \frac{1}{4}P_{c2} + \frac{1}{2}P_{c3}}$$

$$1 - P(c) = P(e)$$

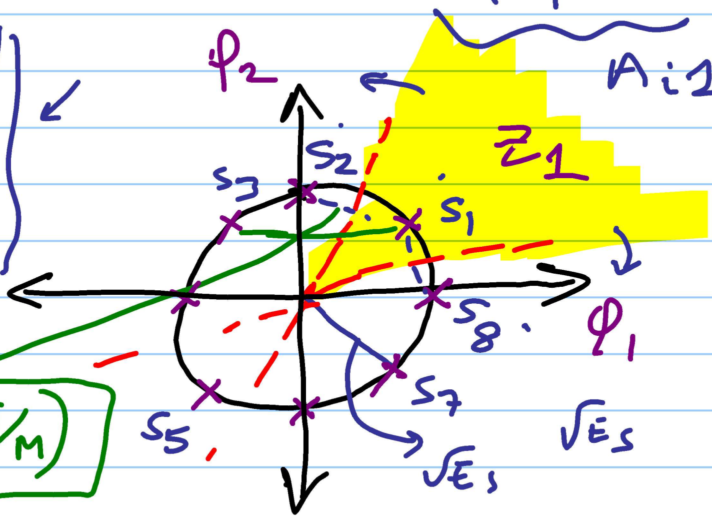
Phase Shift Keying : eg. 8-PSK

$$s_i(t) = g(t)\cos\left[2\pi f_c t + \frac{2\pi}{8}\cdot(i-1)\right], \quad i = 1, 2 \ldots 8 \ (M)$$

$8 \leftarrow M$

$$= \underbrace{g(t)\cos\left(\frac{\pi}{4}(i-1)\right)}_{A_{i1}}\cos 2\pi f_c t + \underbrace{g(t)\sin\frac{\pi}{4}(i-1)}_{A_{i2}}\sin 2\pi f_c t$$

$$\left\{\begin{array}{l} s_1 \to s_2 \\ s_1 \to s_3 \\ \vdots \\ s_1 \to s_8 \end{array}\right.$$



$$\boxed{d_{min} = 2\sqrt{E_s}\cdot\sin(\pi/M)}$$

$r = s_i + n$

upper bound

$\to$ (*) union-bound Argument

$\to$ (*) d_{min} based union bound

$$P(A \cup B \cup C) \leq P(A) + P(B) + P(C)$$

Union Bound on $P(e)$

$$P(e) \rightarrow P[r \notin Z_1 \mid s_1] = P[\mathcal{E}_{12} \cup \mathcal{E}_{13} \cup \dots \mathcal{E}_{18}]$$

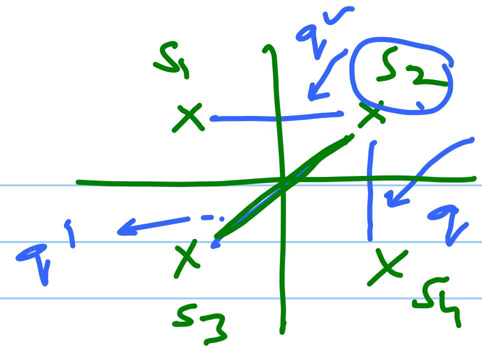
$\mathcal{E}_{12} \rightarrow$ error event that s_1 gets decoded as s_2

$$\leq \sum_{k=2}^{8} P[\mathcal{E}_{1k}] \quad \leftarrow 7 \text{ terms}$$

$$\frac{1}{2}\operatorname{erfc}\left(\frac{d_{min}/2}{\sqrt{N_0}}\right)$$

$$\boxed{P(e) \approx \operatorname{erfc}\left(\sqrt{\frac{E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)} \approx \sum_{k=2,8} P[\mathcal{E}_{1k}] \quad \leftarrow 2 \text{ terms}$$

Example :



$$P(c) = (1-q)^2$$

$$P(e) = 1 - (1-q)^2$$

$$= \underbrace{2q - q^2}$$

$$\underline{\underline{P(e)}} = q + q + q'$$

$$= 2q + q + \varepsilon$$

$$= 3q + \varepsilon$$

$$q' < q$$

$$P(e) = 2q$$

$$P(e) = q$$