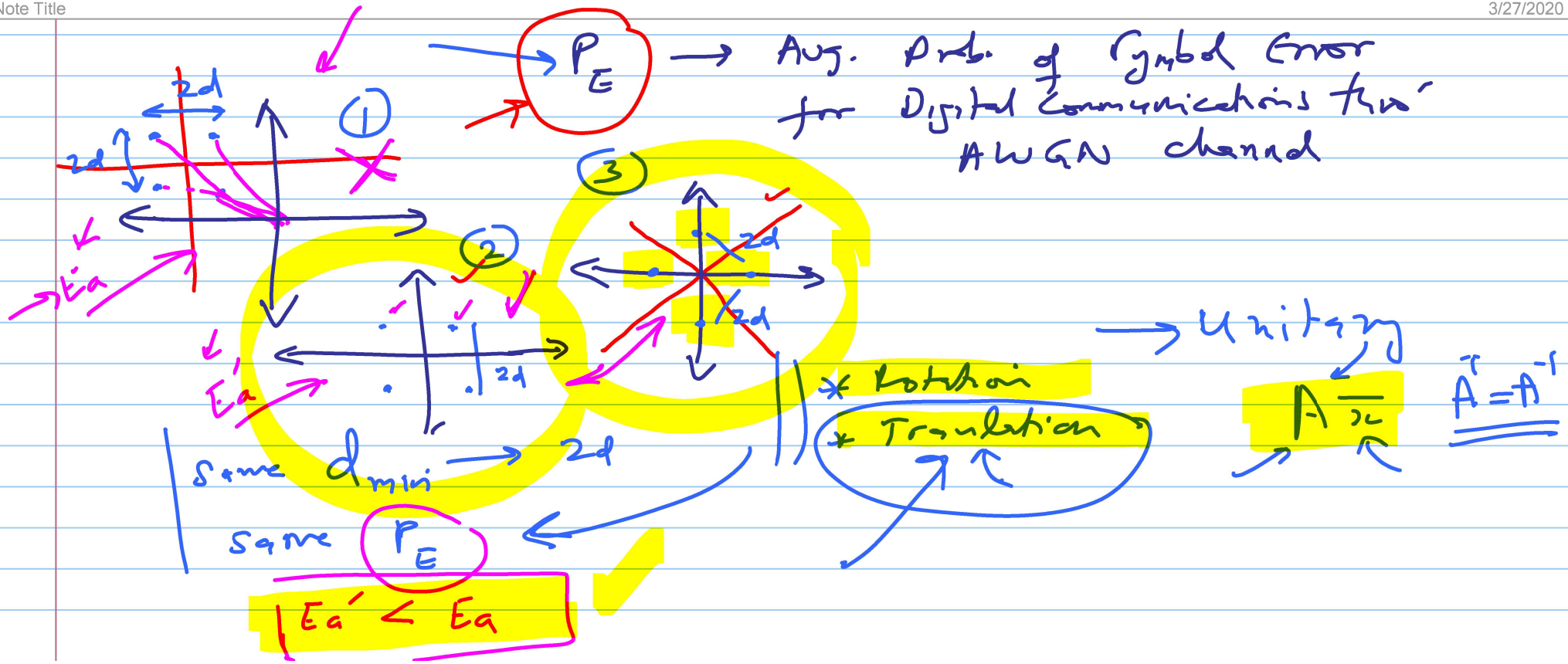
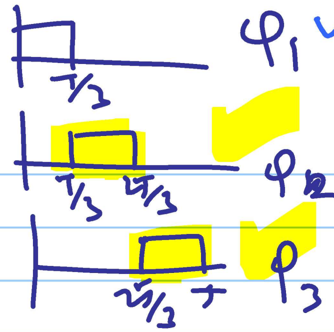
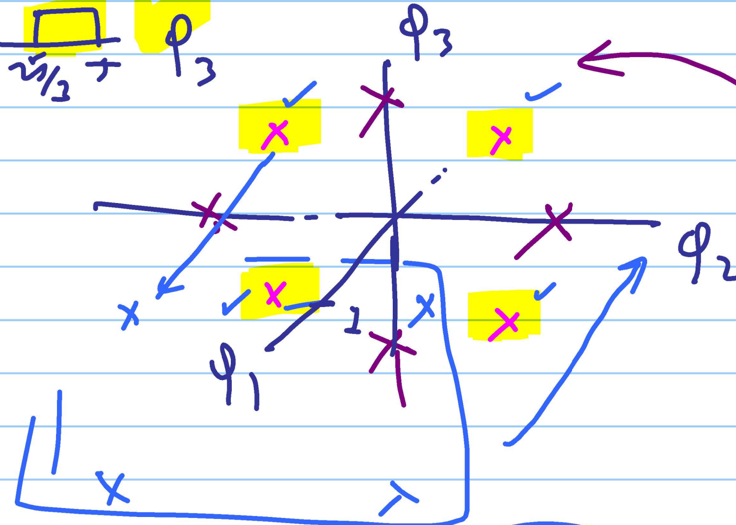
EE4140 **Class # 10** - September 23, 2020

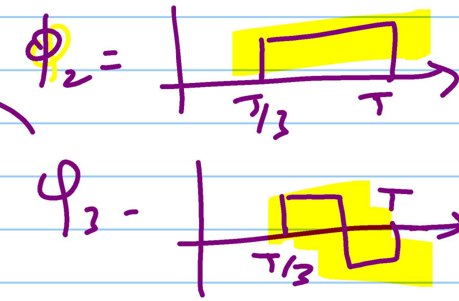
Note Title 3/27/2020

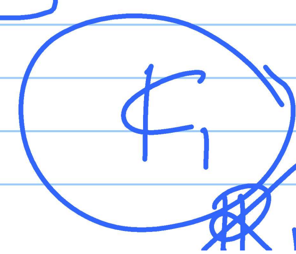




$\bar{s}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$; $\bar{s}_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$; $\bar{s}_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$; $\bar{s}_4 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$



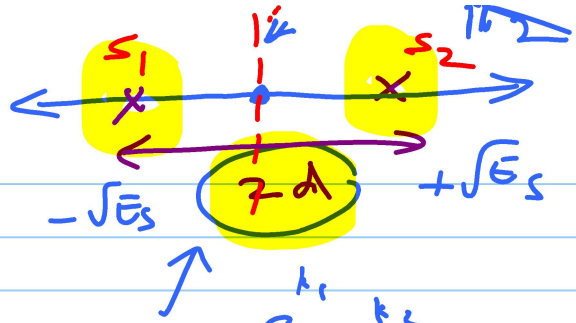




Decision Boundary

$r = s_i + n$ ✓ ✓ ✓ AWGN

$p(s_i) = \frac{1}{M}$ $\forall i, i = 1 \dots M$



Note on Notation

QAM

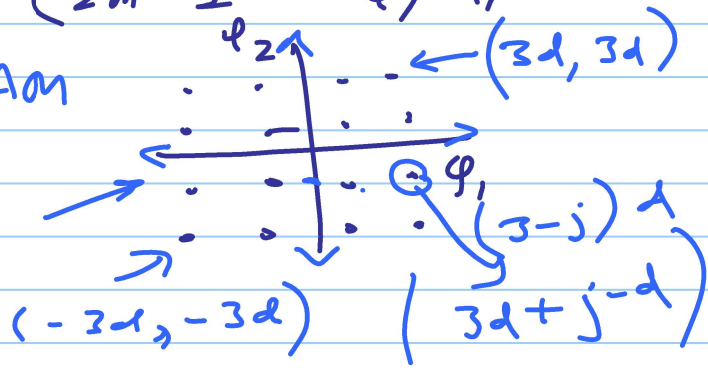
$s_{m_I} = A_{m_I} \sqrt{\frac{E_g}{2}}$, $A_{m_I} = (2m - 1 - M_I)d$, $m = 1 \dots M_I$ (2^{k_I})

$s_{m_Q} = A_{m_Q} \sqrt{\frac{E_g}{2}}$, $A_{m_Q} = (2m - 1 - M_Q)d$, $m = 1 \dots M_Q$ (2^{k_Q})

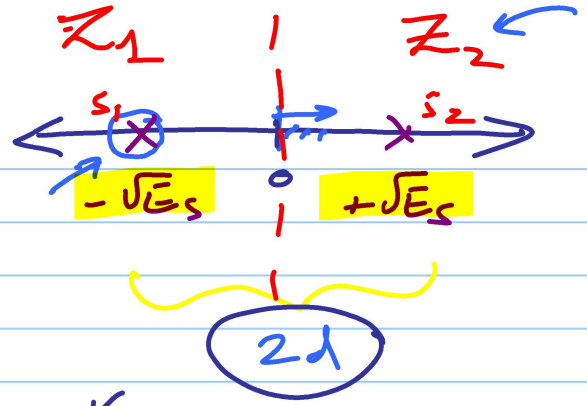
$q = M_I = M_Q$ ✓ ✓ → QAM → "Square" QAM

$A_{m_I} = \{-3d, -d, d, 3d\} = A_{m_Q}$

$2d$ $2d$ $2d$



$$r = s_i + n$$



Let s_1 be transmitted $\rightarrow r = -\sqrt{E_s} + n$

Avg. probability of symbol error.

$$P(e) = P(e|s_1)\,P(s_1) + P(e|s_2)\,P(s_2)$$

$P(e) \rightarrow P_E$

$P(e|s_1) = q = P_1$ $\qquad$ $P(e|s_2) = P_2 = q$

$$\boxed{P(e) = q}$$

$$P_1 \triangleq P(r \in Z_2 \mid s_1 \text{ sent}) = \int_{Z_2} p(r|s_1)\,dr$$

$$= P[r \geq 0 \mid s_1 \text{ sent}]$$

$$= P[n \geq \sqrt{E_s} \mid s_1 \text{ sent}] \rightarrow P[n \geq \sqrt{E_s}]$$

$$P(n \geq \sqrt{E_s}) = \int_{\sqrt{E_s}}^{\infty} f_N(n)\, dn = \int_{\sqrt{E_s}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-n^2/N_0}\, dn$$

$N \sim \mathcal{N}(0, \frac{N_0}{2})$

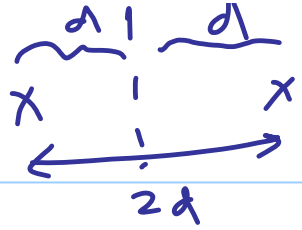
substitution $\beta = n/\sqrt{N_0} \Rightarrow d\beta = \frac{dn}{\sqrt{N_0}}$ $\Big|_{\substack{n=\sqrt{E_s} \\ \beta = \sqrt{E_s/N_0}}}$

$$P_1 = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_s/N_0}}^{\infty} e^{-\beta^2}\, d\beta$$

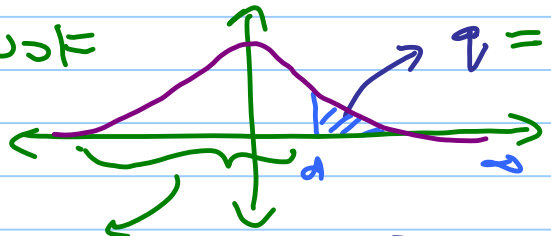
"pairwise" error event probability

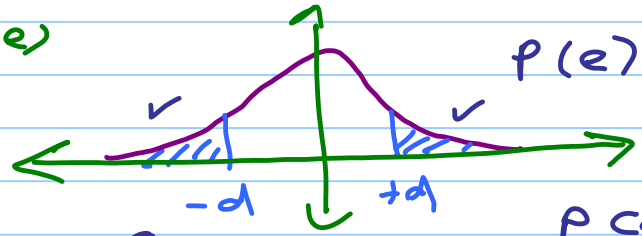
$$= \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_s}{N_0}}\right) = Q\left(\sqrt{\frac{E_s}{N_0/2}}\right)$$

$$P_1 = P_2 = P(e) = q = \begin{cases} \frac{1}{2}\operatorname{erfc}\left(d/\sqrt{N_0}\right) \\ \text{or} \\ Q\left(d/\sqrt{N_0/2}\right) \end{cases}$$



(x) NOT

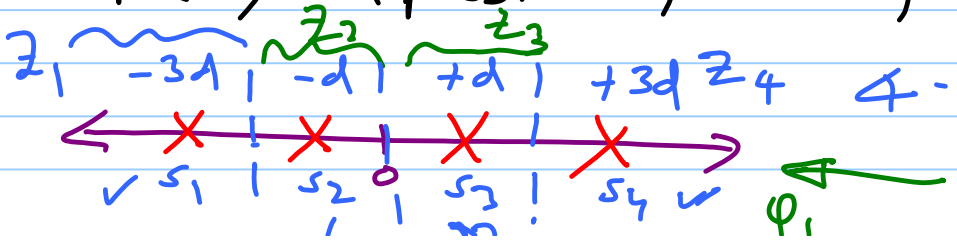




P(c) $\boxed{P(e) = 1 - P(c)}$

Find P(e) expression for the following signal constellation

Ex 1



4-ary PAM $\quad r = s_i + n$

$P(s_i) = 1/4$ $\quad n \sim \mathcal{N}(0, N_0/2)$

$$P(e|s_1) = q = P(e|s_4)$$
$$P(e|s_2) = 2q = P(e|s_3)$$

$$P(e) = \frac{1}{4}q + \frac{1}{4}2q + \frac{1}{4}2q + \frac{1}{4}q$$
$$= \frac{6q}{4} = \boxed{\frac{3q}{2}}$$