

1. [5 marks] We consider raised cosine (RC) pulse shaped transmission. The impulse response of the pulse-shaping continuous time filter  $g(t)$  is given as follows:

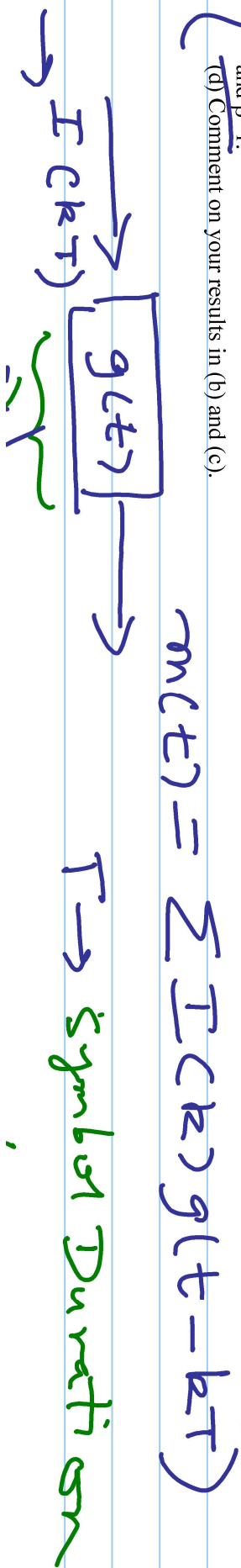
$$g(t) = \text{Sinc}(\pi t/T) \{ \text{Cos}(\pi \beta t/T) / (1 - 4\beta^2 t^2/T^2) \}$$

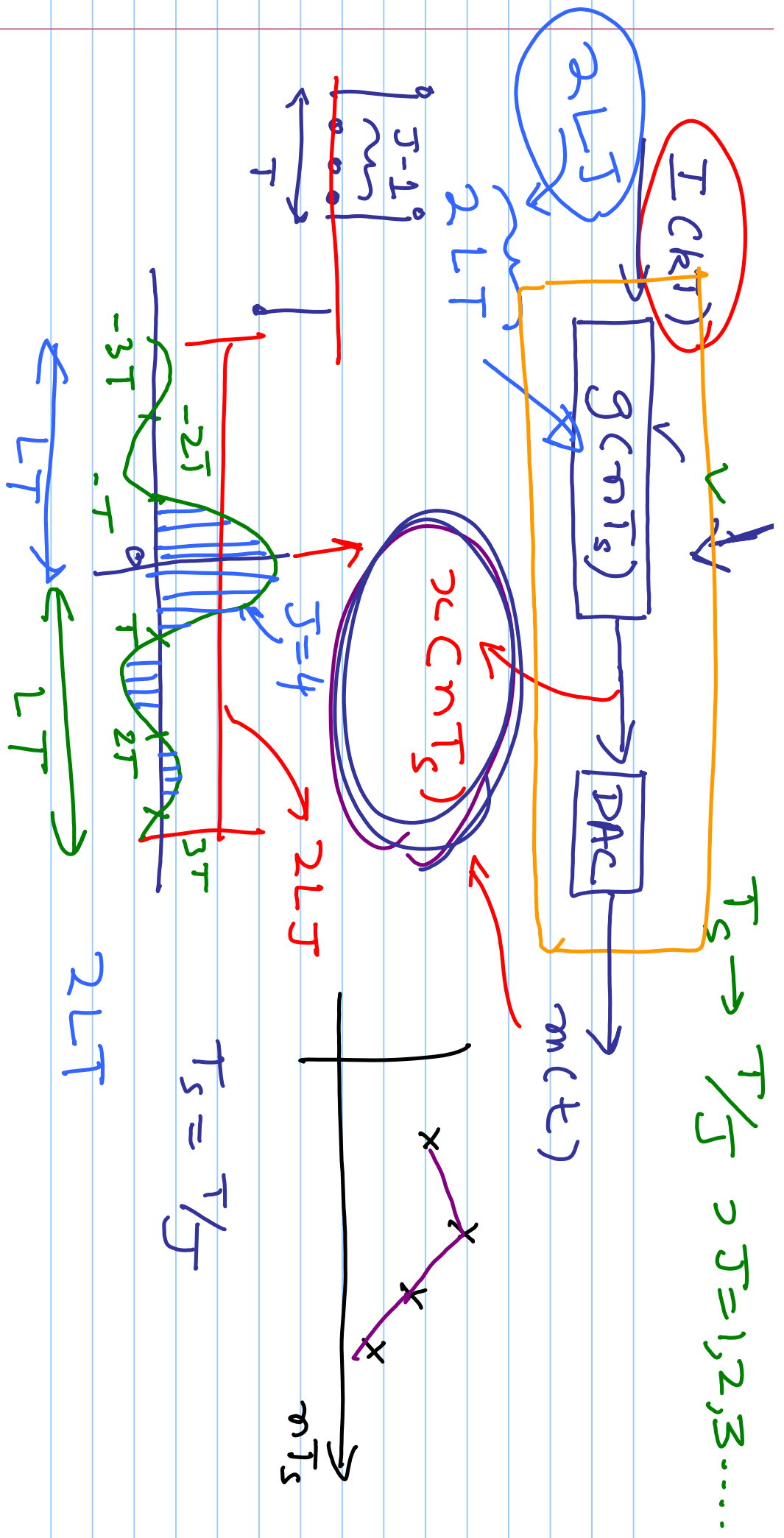
$t = nT_s, T_s = \frac{T}{5}$

where the excess bandwidth factor  $0 \leq \beta \leq 1$  and  $T$  is the symbol duration and  $\text{Sinc}(x) = \text{Sin}(x)/x$ . We use a discrete-time filter  $g(nT_s) = g(n)$  where the sampling rate  $T_s = T/J$ , where  $J$  is a positive integer. Also, the filter is truncated to  $2LT$  symbol durations, i.e.,  $L T$  symbol durations on each side of  $t=0$ , where  $L$  is a positive integer. The input bit (or symbol) stream  $\{k(T)\}$  into this filter will be zero-interleaved with  $J$  zeros in order to create the output sequence  $x(kT_s)$  which is then sent into a DAC.

Chose your own 16-bit random sequence where  $I(k) \in \{+1, -1\}$  in order to create a BPSK signal. Assume all the bits before and after this 16-bit pattern are zeros. You are to plot the output sequence (samples)  $x(kT_s)$  using both "symbol-plot" as well as join them using "line-plot" using Matlab.

- (a) Plot as Fig.1,  $x(kT_s)$  for  $J=4, L=2$ , and  $\beta=0$ .
- (b) Repeat (a) and include in Fig.1 another plot with the change  $L=4$ .
- (c) Plot as Fig.2, three different plots of  $x(kT_s)$  where all of them have  $J=8, L=2$ , but with three different excess bandwidth factors, namely,  $\beta=0, \beta=0.5$ , and  $\beta=1$ .
- (d) Comment on your results in (b) and (c).





2. [15 marks] In this question, we compare the theoretical probability of symbol error  $P(e) = P_s$  and bit error  $P_b$  with computer simulated symbol or bit error rate (SER or BER), for a few popular linear modulation schemes. The approximation on  $P_s$  using bounds will also be studied.

BER/SER

(a) For the same energy per bit,  $E_b$ , compute and plot on the same figure (Fig-1) the  $P_s$  of BPSK, QPSK, and 16-QAM signals. Assume the complex base-band AWGN measurement model  $r(k) = I(k) + jv(k)$ , where  $I(k)$  is uncorrelated with the white Gaussian noise  $v(k)$ . For complex signals (QPSK and 16-QAM) the noise will be complex with variance  $N_0/2$  per dimension. Vary the ratio  $10 \log_{10}(E_b/N_0)$  in 2dB steps from 0dB to 10dB, by changing the noise variance and plot against  $\log_{10}(P_s)$ , for all the 3 signals. Chose  $E_b=1$  for all the signals. Note that the log of the error probability is plotted on the y-axis. The  $Q(\cdot)$  function or the  $\text{erfc}(\cdot)$  function can be evaluated using Matlab.

(b) Assuming Gray coding, plot  $10 \log_{10}(E_b/N_0)$  versus  $\log_{10}(P_b)$  for the 3 signals. Add these curves also to Fig-1, and label neatly.

(c) For the QPSK signal set, compute using the following approximations to the symbol error probability. Plot each of these values of  $P_s$  for the range of SNRs from 0dB to 16dB in the same figure. Call it Fig-2.

(c1) Union bound using all the pairwise symbol errors

(c2) Union bound using only the nearest neighbours

(c3) In part (c2) replace the  $\text{erfc}(\cdot)$  function with the Chernoff bound and add this also to Fig-2.

(d) Now, we wish to simulate the symbol error rate (SER) for QPSK. For generating the transmit symbols, use uniform random variable (rv) between (0,1). If the rv takes a value  $X$  between 0 and 0.5, it is mapped to say  $-d$ ; else, it is mapped to  $+d$ , where  $2d$  is the distance between the symbols. We will need 2 rv's, one of the real part and one for the imaginary part. Chose "d" to ensure  $E_b=1$ . Generate  $10^5$  symbols to measure the SER over the same range of SNRs as in part (c). Plot your results back in Fig-2. Comment.

(e) Repeat part (c) above for the 16-QAM signal set. Call this Fig-3.

(f) Repeat part (d) for the 16-QAM signal set. Chose the rv to symbol mapping appropriately and explain how you did it. Plot this result back in Fig-3.

(g) With Gray-coding done, compare the simulated bit error rate (BER) of QPSK and 16-QAM. (Hint: while you are measuring the SER in parts (e) and (f), you can also compute the BER for both the modulations). Plot these two results together in Fig-4, again for  $E_b=1$  in both cases. Note that for computing BER, you will

$$10^{-3}$$

$$\log_{10} (10^3) = 3$$

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Instructions: Submit only a single PDF file for the report. Name the file as "rollnumber-assignment1-report.pdf". Your working code, properly commented, **must be included** as Annexure-1 to the PDF file, and also separately sent by email to the TAs. Your working code can be named "rollnumber-assignment1-code.m". Please see other instructions, if any, in the WhatsApp group.

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$\cancel{SER}$   
 $P(\infty)$   
 $P_s = P_s$   
 $\log_{10} P_s$

