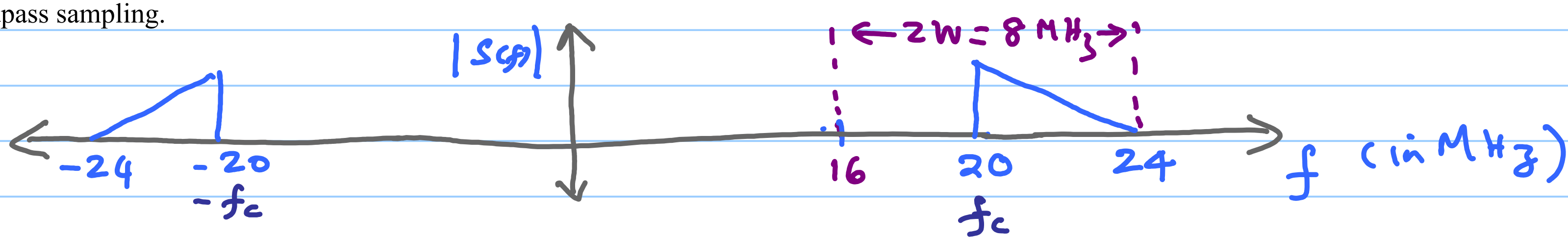
1.[1.5+4.5=6 marks] A real low-pass signal s(t) of <u>one-sided</u> bandwidth W=4MHz is sent as an upper-sideband only SSB-SC signal with magnitude spectrum as shown below. The receiver uses bandpass sampling.



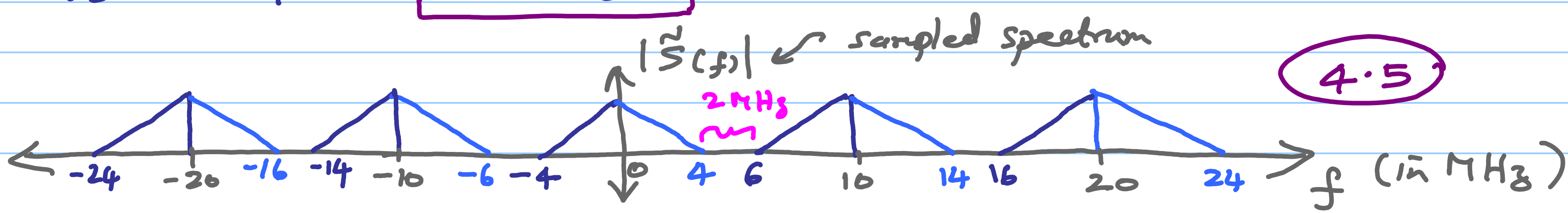
(a) Determine the least sampling rate $f_S = 1/T_S$ required in MHz.
(b) Make a neat plot of the magnitude spectrum |S'(f)| of the sampled sequence $s(kT_S)$ between 24MHz and -24MHz.

(a) $2W = 8$ MHz does not divide $f_c = 20$ MHz ;

The largest integer in $f_c/2W$ is given by $\left\lfloor \frac{20}{8} \right\rfloor = 2$;

Hence, $f_s = 20/2 = \boxed{10 \text{ MHz}}$; 1.5

(b)



4.5

2.[1+6+1=8 marks] A real bandpass QCM signal with carrier frequency f_c=200KHz has the two message signals $m_1(t)$ and $m_2(t)$ of one-sided bandwidth of W=5KHz each. Let $S(f)$, $M_1(f)$, and $M_2(f)$ denote the (complex) Fourier transforms of $s(t)$, $m_1(t)$ and $m_2(t)$, respectively. Given $M_1(f=2\text{KHz}) = a + jb$ and $M_2(f=2\text{KHz}) = c + jd$ where as usual, j = sqrt(-1), determine the following in terms of a, b, c, and d. The number inside the brackets can be assumed to be in KHz.

(a) $M_1(-2)$ and $M_2(-2)$
(b) $S(202)$ and $S(198)$

(a) Given $M_1(2) = a + jb$, and $M_2(2) = c + jd$ $\quad j = \sqrt{-1}$

$\boxed{M_1(-2) = a - jb}$; 0.5 $\quad \boxed{M_2(-2) = c - jd}$; 0.5

(b) $$S(f) = \frac{M_1(f-f_c) + M_1(f+f_c)}{2} + \frac{M_2(f-f_c) - M_2(f+f_c)}{2j}$$

$$\therefore S(202) = \frac{a+jb}{2} + \frac{c+jd}{2j} = \frac{1}{2}\left((a+d) + j(b-c)\right) \rightarrow 3$$

$|||^{ly}$

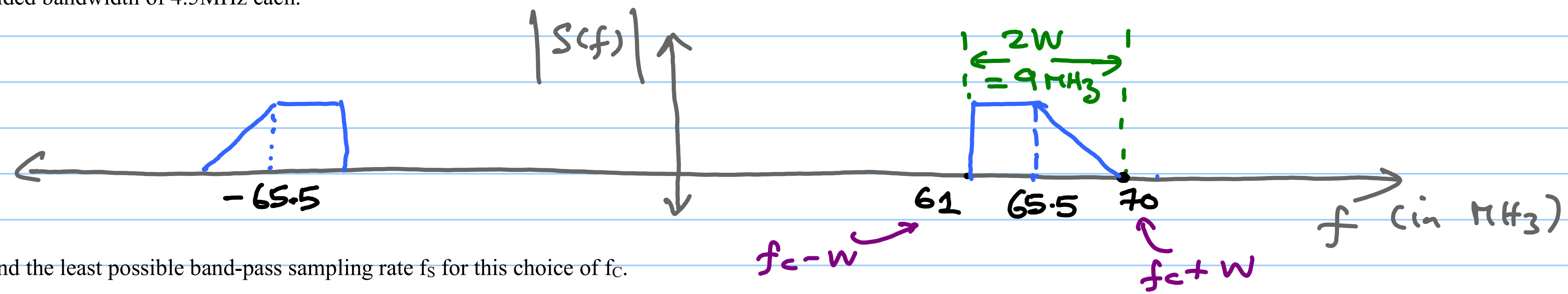
$$S(198) = \frac{a-jb}{2} + \frac{c-jd}{2j} = \frac{1}{2}\left((a-d) - j(b+c)\right) \rightarrow 3$$

Note: $S(198) \neq S^*(202)$ since there is no even symmetry around $f_c = 200$ kHz for QCM signals

(c) $S(-198) = S^*(198) \leftarrow$ since $s(t)$ is real & $s(t) \rightleftharpoons S(f)$

$\therefore S(-198) = \boxed{\frac{1}{2}\Big((a-d) + j(b+c)\Big)}$ -1-

3.[3+3=6 marks] Consider a different QCM signal with magnitude response as below. *Note*: From the information given below, note that both the low-pass message signals have one-sided bandwidth of 4.5MHz each.



(a) Find the least possible band-pass sampling rate f_S for this choice of f_C.

(b) If you were allowed to modify (or design) the carrier frequency between 65MHz and 70MHz given these message signals, what would you choose as f_C to minimize the band-

(a) For QCM, check if $\underline{f_c+W}$ or $\underline{f_c-W}$ gives the least f_s

For $4W = 9\times 2 = 18$ MHz

$$\left\lfloor \frac{70}{18} \right\rfloor = 3 \quad (f_c+W) \qquad\qquad \left\lfloor \frac{61}{18} \right\rfloor = 3 \quad (f_c-W)$$

$$\Rightarrow f_{s,1} = \frac{70}{3} = 23.33 \text{ MHz} \qquad f_{s,2} = \frac{61}{3} = \boxed{20.33 \text{ MHz}}$$

-3-

Since $f_{s,2}$ is smaller than $f_{s,1}$, choose $f_{s,2}$

(b) → Note that multiples of $4W = 18 \longrightarrow 36, 54, \underline{\underline{72}}, 90$ MHz

→ As f_c can be picked between {65 MHz to 70 MHz}, the least sampling rate of $f_s = 4W$ we will get only if f_c+W or f_c-W is $\underline{\underline{72}}$ (from the set above)

$\Rightarrow f_c = 67.5$ MHz & $f_c + 4.5 = 72$ MHz is the only choice

-3-

$$\boxed{f_c = 67.5 \text{ MHz}}$$