Lesson #4 (C1 -- Feb. 14, 2022)

Note Title XXXXXXXX

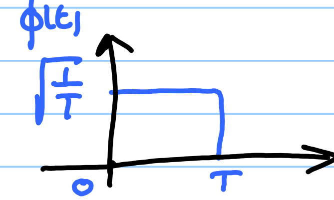
Tx – Rx based Measurement Model

(*) "One shot" Communication

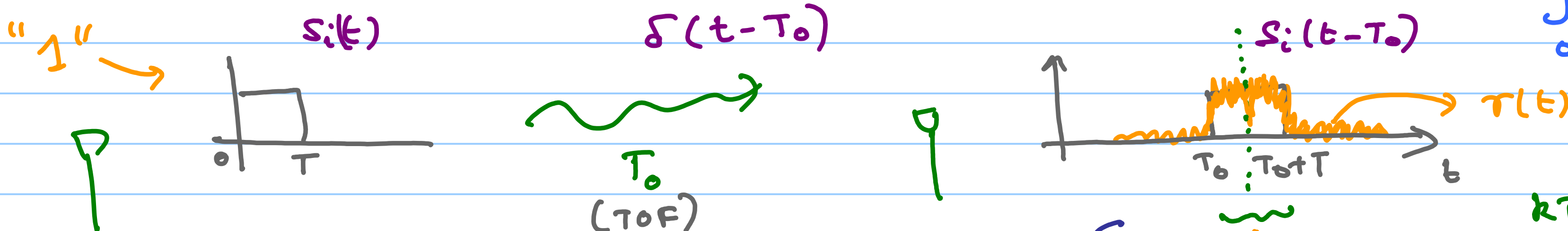
"1" → √E Joules → $\sqrt{E}$ → $s_1(t) = \sqrt{E}\,\phi(t)$

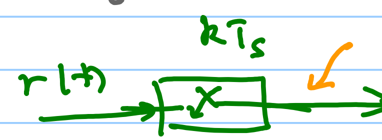
"0" → E Joules → $-\sqrt{E}$ → $s_0(t) = -\sqrt{E}\,\phi(t)$

basis fn: $\phi(t)$

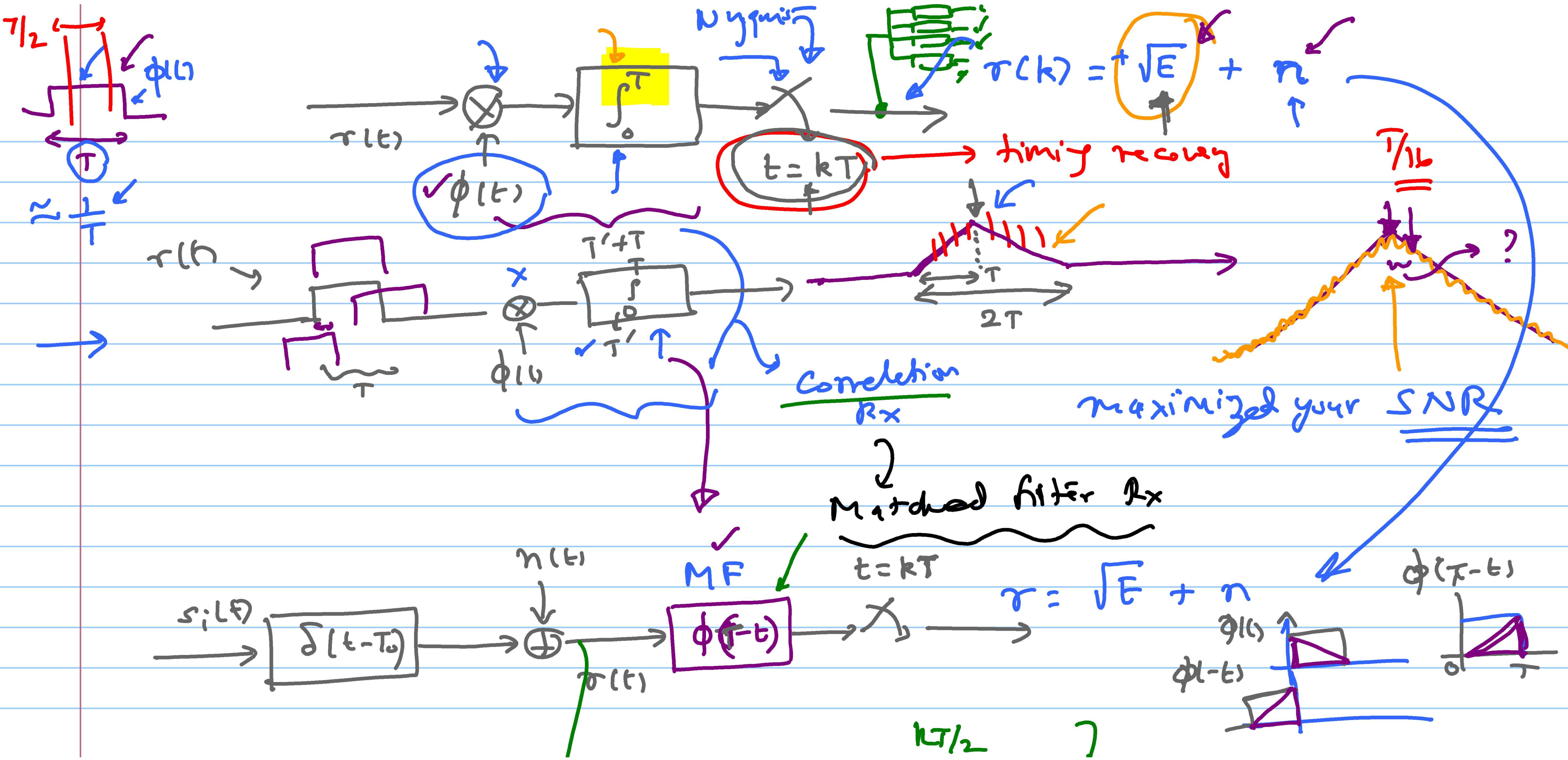


$$\int_0^T \phi^2(t)\,dt = 1$$

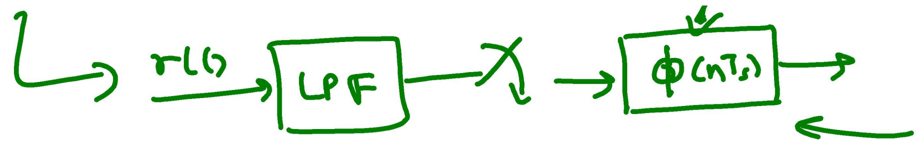




$$r(t) = s_i(t-T_0) + n(t)$$



T/2
φ(t)
T
≈ 1/T
r(t)
φ(t)
Nyquist
r(k) = √E + n
t = kT
timing recovery
T/16
?
r(t)
T'+T
T'
φ(t)
T
2T
Correlation Rx
maximized your SNR
Matched filter Rx
n(t)
MF
t = kT
r = √E + n
φ(T-t)
φ(t)
φ(-t)
s_i(t)
δ(t-T_0)
r(t)
φ(T-t)
0
T
kT/2



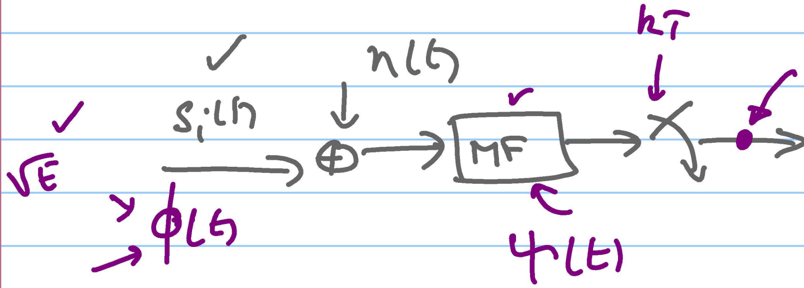
(*) Continuos Comm.

$$\rightarrow s(t) = \sum_{k=-\infty}^{+\infty} I(k)\,\phi(t-kT)$$

$$I(k) = \begin{cases} \sqrt{E}, & \text{"1"} \\ -\sqrt{E}, & \text{"0"} \end{cases}$$

$$r(k) = \begin{cases} +\sqrt{E} + n & \text{"1"} \\ -\sqrt{E} + n & \text{"0"} \end{cases}$$

$k T^{th}$



sig. comp. → $\left| \int \sqrt{E}\cdot\phi(t)\,\psi(t)\,dt \right|^2$

noise power → $E\left[\left(\int n(t)\cdot\psi(t)\right)^2\right]$ "white"

$$\max_{\phi(t)} \text{SNR} \triangleq \frac{E \cdot \left| \int \phi(t)\varphi(t)\,dt \right|^2}{\frac{N_0}{2} \left| \int \phi(t)\,dt \right|^2} \leq$$

$$E[n(t)\,n(\tau)] = \frac{N_0}{2}\,\delta(t-\tau)$$

Cauchy-Schwartz Inequality

$$\leq \int |\phi(t)|^2\,dt \int |\varphi(t)|^2\,dt$$

$$\varphi(t) = \phi(-t)$$

(*) Quick Look at LIGO

cont. time → disc. time

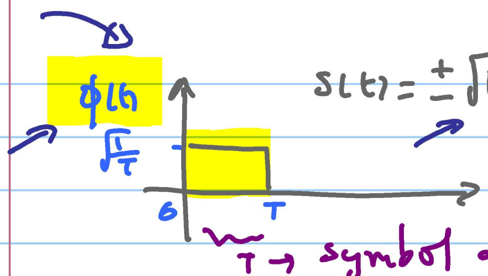
$$r(t) = s_i(t) + n(t)$$

$$\sqrt{E}\,\phi(t) \qquad -\sqrt{E}\,\phi(t)$$

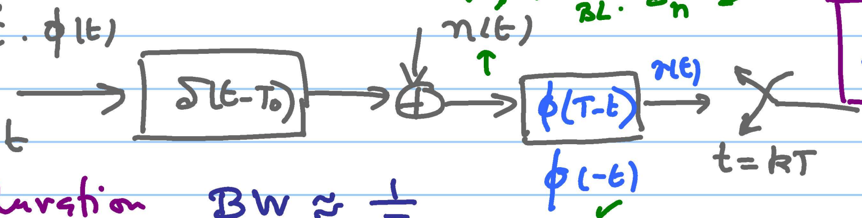
$$r = \begin{cases} \sqrt{E} + n \\ -\sqrt{E} + n \end{cases}$$

EE3007 Jan-Apr, 2022

Lesson #4 (C2 -- Feb.15, 2022)



$s(t) = \pm\sqrt{E}\cdot\phi(t)$ ← E Joules



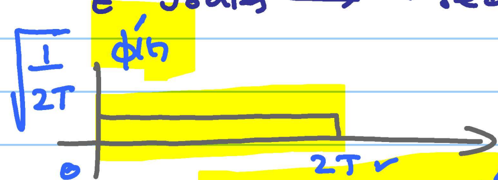
$\int \phi^2(t)\,dt = 1$

$r = \pm\sqrt{E} + n$ ← σ_n^2, $\mathcal{N}(0, \sigma_n^2)$

T → symbol duration BW $\approx \frac{1}{T}$

$R \to W \cdot \log_2(1 + SNR)$
bit/s Hz

(*) E Joules → T secs ⇒ Power = $\frac{E}{T}$ watts ← R bit/sec (T)
P @ Tx

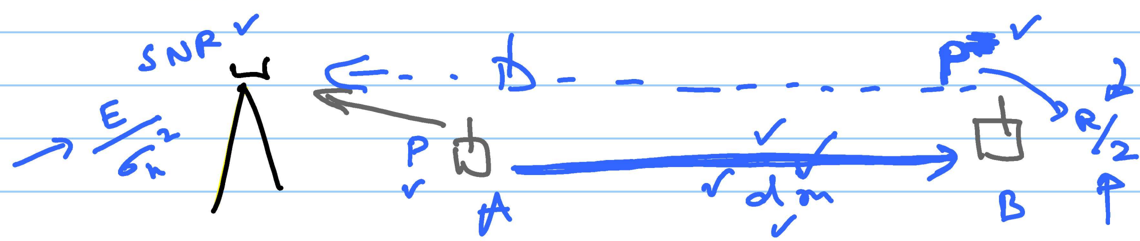


$s(t) = \pm\sqrt{E}\,\phi'(t)$

Energy per symbol → E';

Power P' = $\frac{E}{2T}$ watts ← $\frac{R}{2}$ bits/sec (2T)

$P \propto I^2 \cdot R$



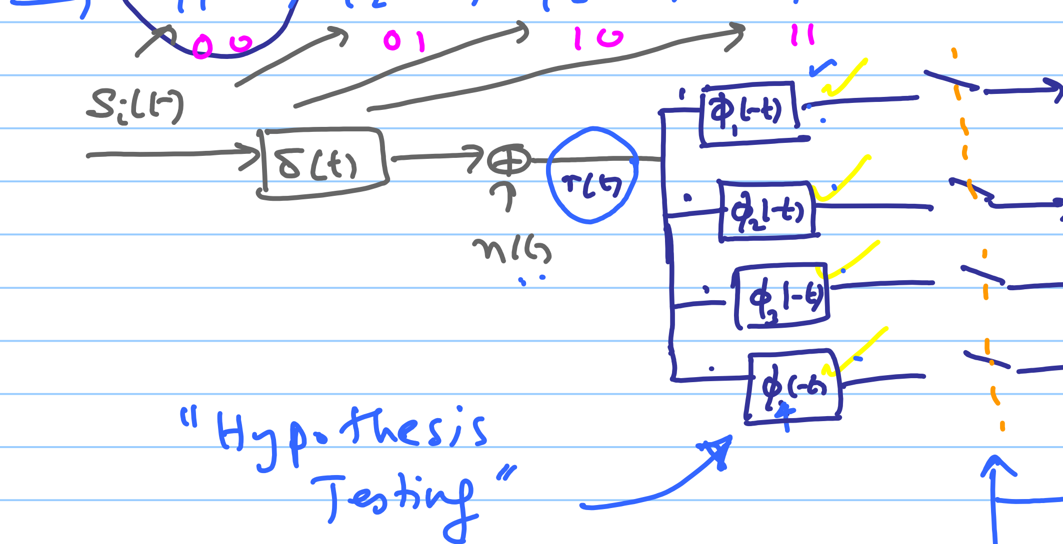
$\frac{E}{bit} \cdot \frac{bit}{sec} = \frac{E}{sec}$ ← P

(*) Orthogonal signaling

$\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t)$

00 01 10 11

$$\int_0^T \phi_i(t)\,\phi_j(t)\,dt = \begin{cases} 1, & i=j \\ 0, & i\neq j \end{cases}$$

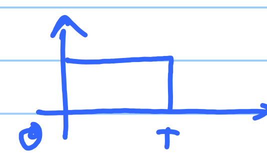


$z_1 = \sqrt{E} + n_1$. 1.1

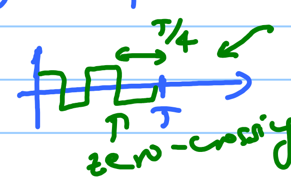
$z_2 = n_2$ 1.1

$z_3 = n_3$. 1.1

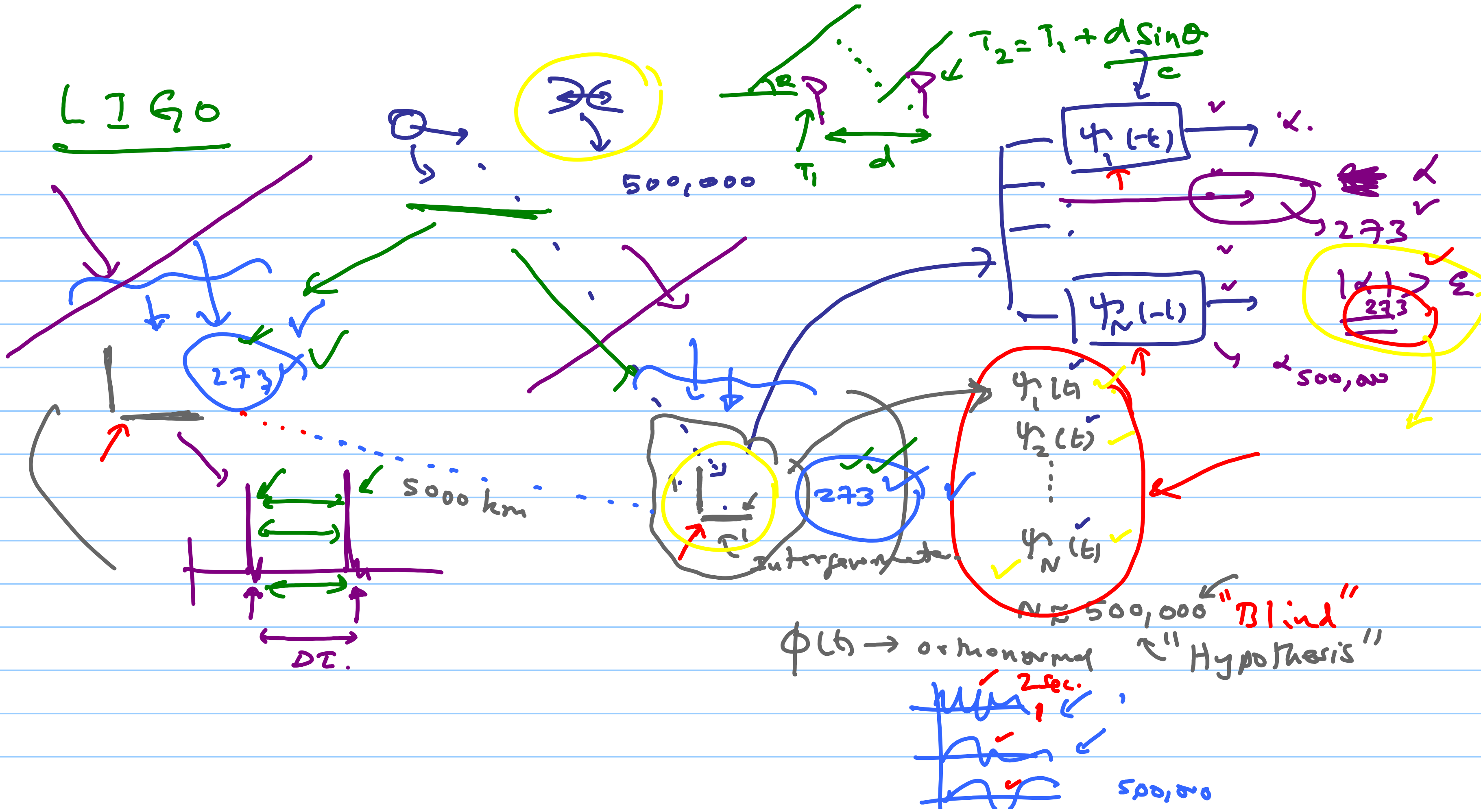
$z_4 = n_4$ 1.1



$BW \approx \frac{1}{T}$



$BW \approx \frac{4}{T}$


LIGO
500,000
T_1
d
$T_2 = T_1 + \frac{d \sin\theta}{c}$
$\psi_1(-t)$
$\psi_N(-t)$
273
273
273
273
500,000
5000 km
ΔT.
Interferometer
$\psi_1(t)$
$\psi_2(t)$
$\psi_N(t)$
N ≈ 500,000 "Blind"
$\phi(t) \rightarrow$ orthonormal
"Hypothesis"
2 sec.
500,000