EE3007 Jan-Apr, 2022

Lesson #2 (C1 -- Jan. 24, 2022)

Note Title XXXXXXXX

$R_N(\tau)$ $R_S(\tau)$ $R_S(t;\tau)$

$E[n(t)\,n(t-\tau)]$

$E(\cdot)$

low pass signal $m(t)$

real

"energy" too

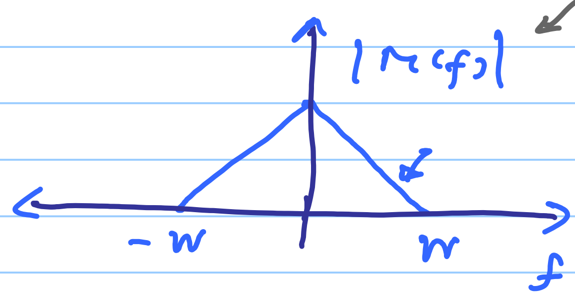
$\int_{-\infty}^{\infty} m^2(t)\,dt < E_g$

Band-Pass Sampling Theorem

$|M(f)|^2$

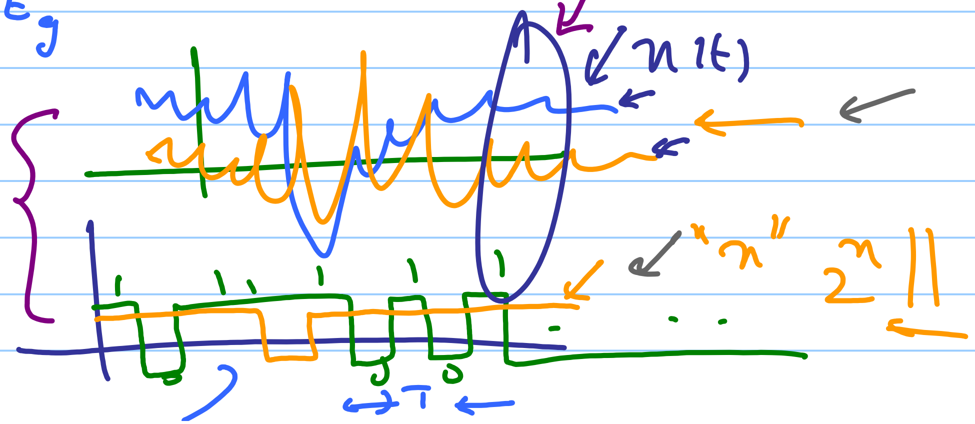
$R(\tau) \rightleftharpoons S(f)$

acf PSD



"Random Noise" $n(t)$

Digitally modulated Signal $s(t)$



"Pulse . shape"

$S(f) \rightarrow \frac{|G(f)|^2}{T} \leftarrow$ "Pulse . shape"

$\approx \frac{1}{T}$ Hz

Group 1

DSB - SC

Double Sideband Suppressed Carrier

SSB - SC ✓

Single

Full - AM ✓ Carrier

VSB ✗ Vestigial SB ✓

:

Group 2

QCM ✓ ✓ $\rightarrow$ PSK, ASK, QAM

Quadrature Carrier Multiplexing

Real

$m(t) \Rightarrow |M(f)|$

$W = 1$ MHz

$f_c = 1$ GHz

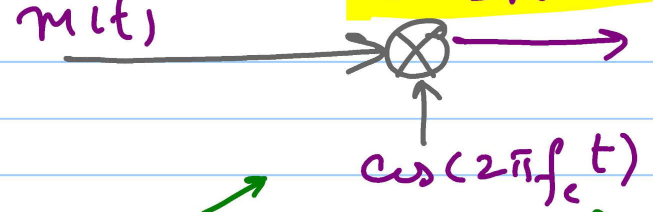
100 KHz $\rightarrow$ 200 kHz

80 MHz – 110 MHz

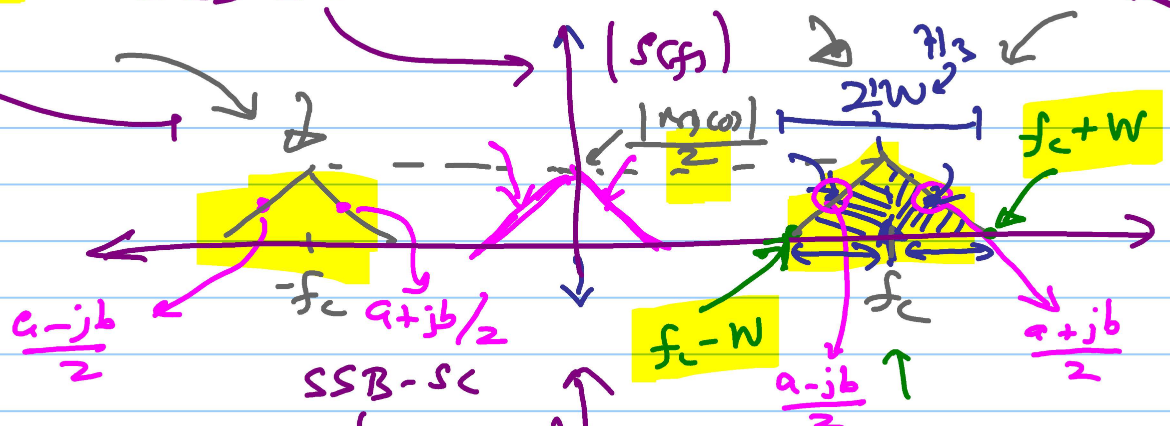
$\boxed{f_c \gg W}$

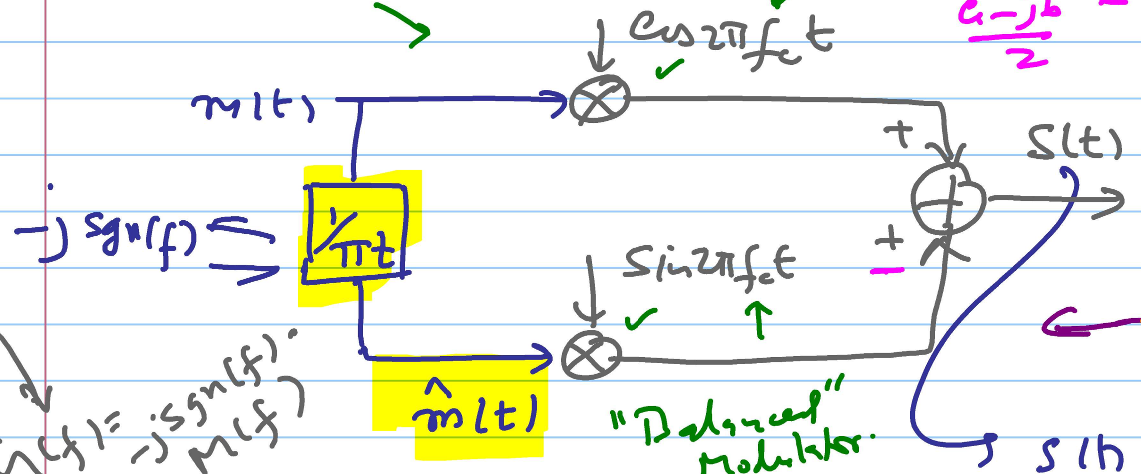
DSB-SC

$s(t) = m(t)\cos 2\pi f_c t$



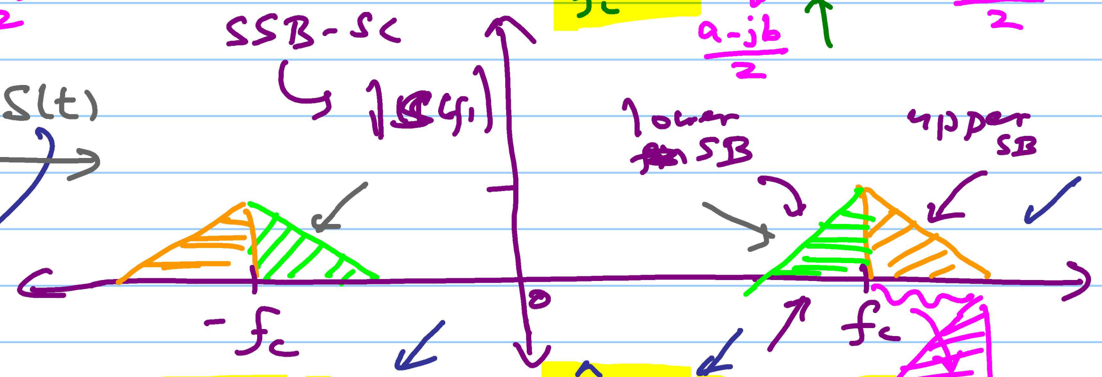
$M(f) * \dfrac{\delta(f-f_c)+\delta(f+f_c)}{2}$





$\hat{M}(f) = -j\,\mathrm{sgn}(f)\, M(f)$

SSB-SC



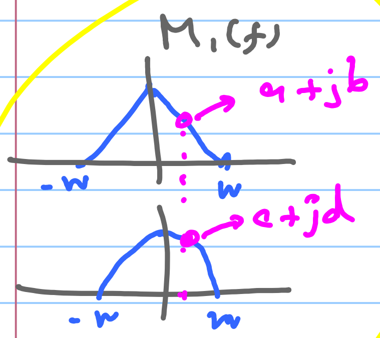
$s(t) = m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t$

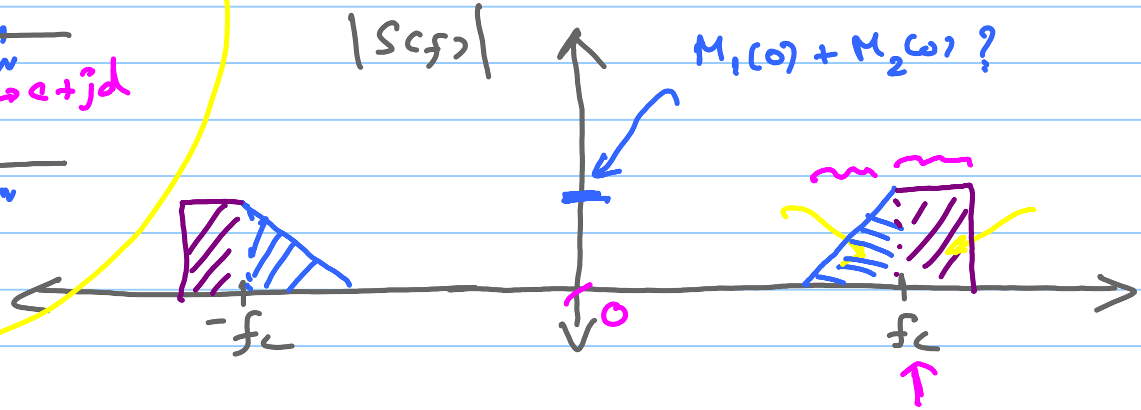
Lesson #2 (C2 -- Jan. 28, 2022)

Recall, Quadrature Carrier Multiplexing (QCM)

$$s(t) = \underbrace{m_1(t)\cos 2\pi f_c t}_{\frac{M(f-f_c)}{2} \quad \frac{M(f+f_c)}{2}} + m_2(t)\,\underbrace{\sin 2\pi f_c t}_{\frac{\delta(f-f_c)-\delta(f+f_c)}{2j}}$$

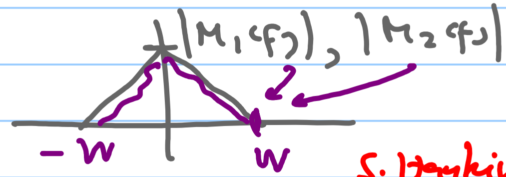
ex:





DSB-SC $\rightarrow m_2(t) = 0$

SSB-SC $\rightarrow m_2(t) = \hat{m}_1(t)$

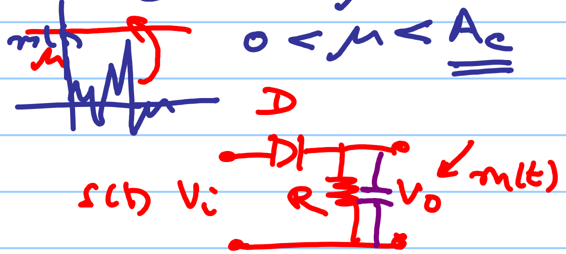


Full-AM ← S. Haykin

$$\boxed{s(t) = \stackrel{A_c}{m(t)}\cos 2\pi f_c t + \stackrel{A_c}{\mu}\cdot\cos 2\pi f_c t}$$

$$= A_c\,(m(t) + \mu)\cos 2\pi f_c t$$

$0 < \mu < A_c$



Recall: $m(t) \xrightarrow{\quad} \times \xrightarrow{\quad} m[kT_s] \to m[k]$, sampled at kT_s

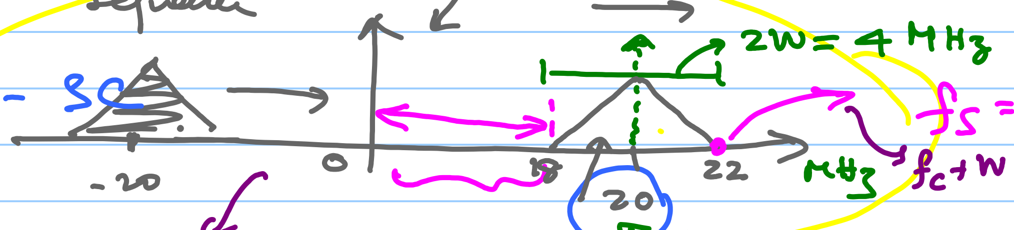
$m(t) \rightleftharpoons M(f)$

$$\mathcal{F}\left[m(k)\right] = \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} M\left(f - \frac{k}{T_s}\right)$$

FT of the sampled sequence

$\leftarrow f_s = \frac{1}{T_s} \geq 2W$

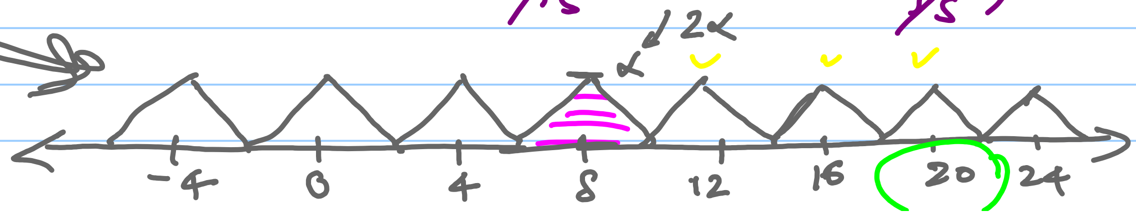
(*) DSB - SC



$f_s = 44$ MHz

$s(t) \to$

$\to f_s \cancel{\frac{1}{T_s}} \sum S\left(f - \cancel{\frac{kf_s}{T_s}}\right)$



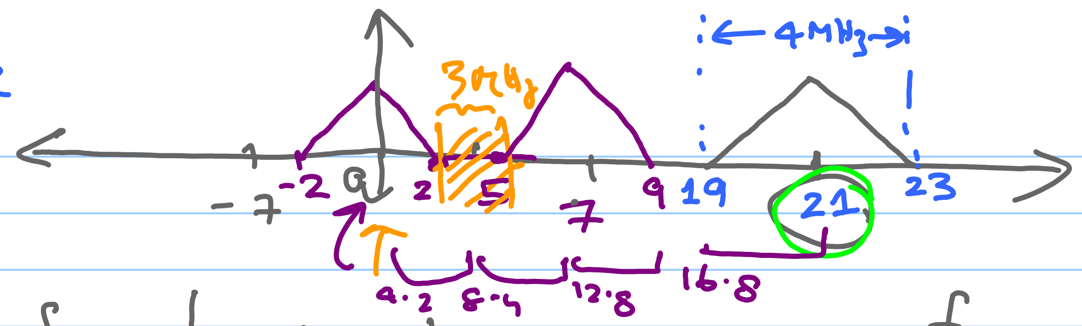
$f_c = 20$ MHz

$\left\{ \frac{f_c}{2W} \right.$ ← Integer.

$\boxed{f_s = 2W \text{ Hz} = 4 \text{ MHz}}$

1

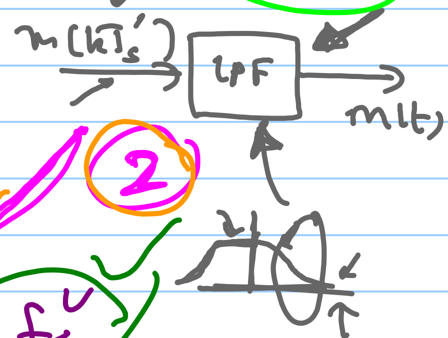
(*) DSB - SC



→ $f_s = 7\,MHz$

Best ?

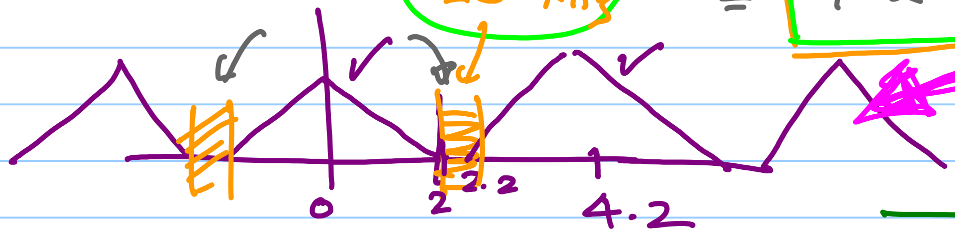
$W = 2\,MHz$



$$\left\lfloor \frac{f_c}{2W} \right\rfloor = \left\lfloor \frac{21}{4} \right\rfloor = 5 \; ; \quad f_s = \frac{f_c}{5} \; ; \quad = \boxed{4.2\,MHz} \; ;$$

200 kHz

- SSB-SC
- Full-AM



$2W$ f_c f_s

(*) In the above example, take $f_c = 19\,MHz$ & $2W = 4\,MHz$

(*) Show that $f_c = \underline{19.99}\,MHz$ will give (approximately) the largest sampling rate

DSB-SC

$s(t) = m(t)\cos(2\pi f_c t)$

$t \to nT_s$

$f_c \leftrightarrow T_s$

$f_c \leftrightarrow 2W$

$m(nT_s)$

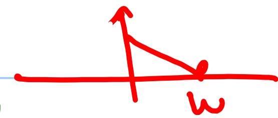
$\boxed{\frac{1}{T_s} = 2W}$

$k \cdot \left(\frac{1}{T_s}\right)$

$f_s \to \boxed{\frac{1}{T_s} \geq 2W}$

f_c

$\cos(2\pi k) \to 1$



$m(nT_s) = m\left(\frac{t}{2W}\right)$

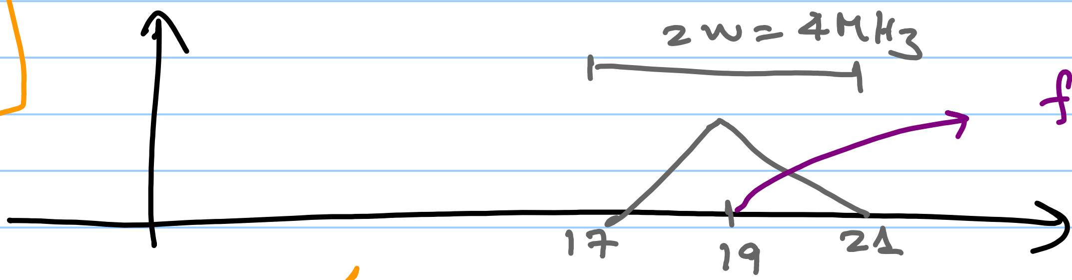
$\left[\frac{209}{2W}\right]$ MH

$= 50$

$\frac{201}{50} \approx 4$ MHz

$K = \left\lfloor \frac{f_c}{2W} \right\rfloor$

$f_s = \frac{f_c}{K}$

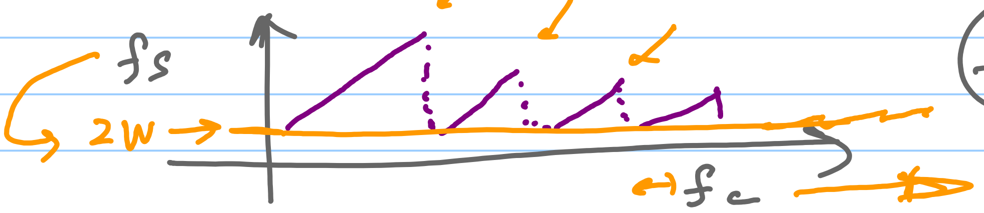


$f_s = 4.75$ MHz

DSB-SC

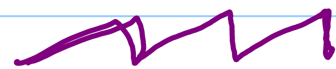
$2W = 4$ MHz

f_c	f_s
20	4.0 MHz
21	4.2
19	4.75 MHz
19.99	≈ 4.99 MHz



2W

DSB-SC

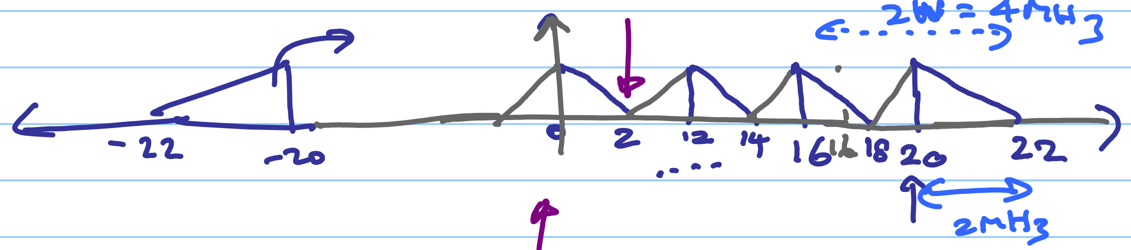


EE3007 Jan-Apr, 2022

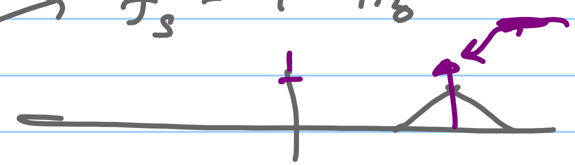
Lesson #2 (C3 -- Jan. 31, 2022)

* what about SSB-SC & Full-AM ?

$\frac{20}{4}$



$f_s = 4$ MHz



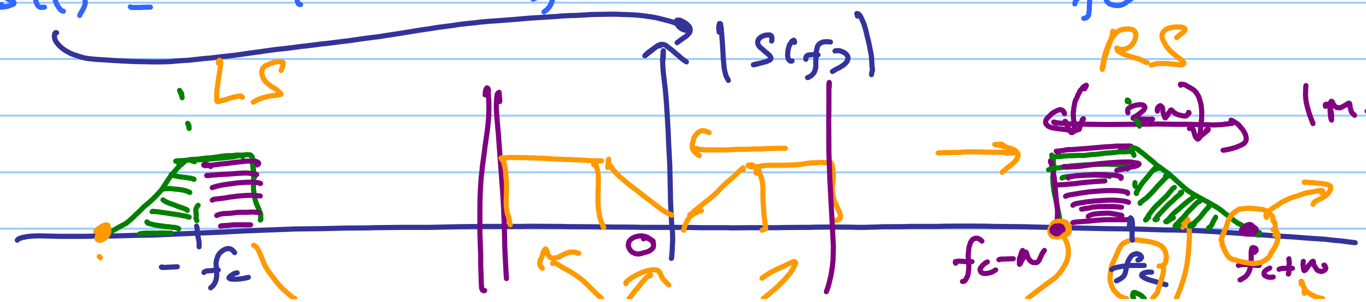
$f_c \leftrightarrow 2W$

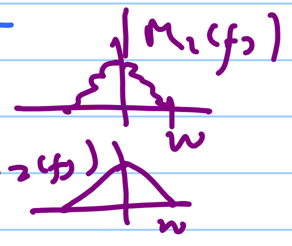
$\rightarrow \; s(t) = m(t) \cos 2\pi f_c t \pm \hat{m}(t) \sin 2\pi f_c t$

$f_s = 2W \rightarrow f_s = 2W' > 2W$

PCM

$s(t) = m_1(t) \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t$



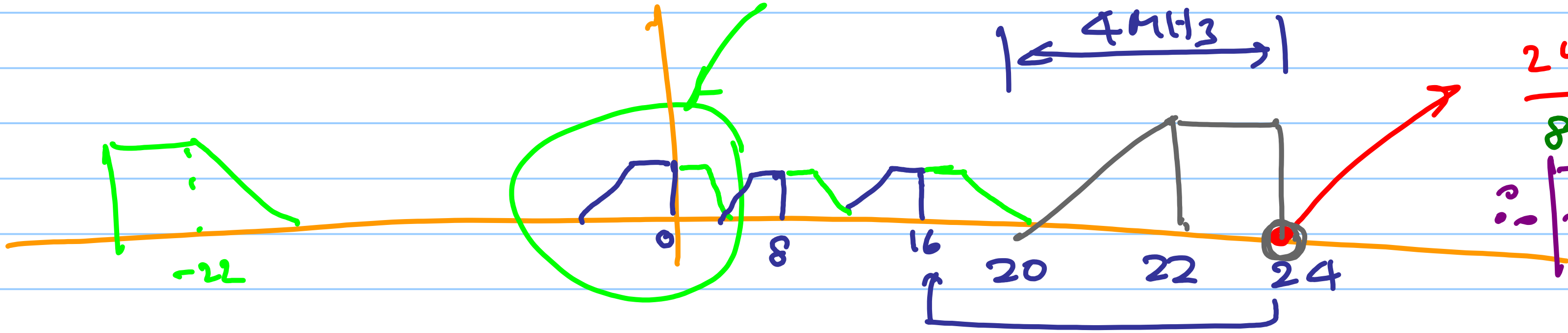


$f_c + W \longleftrightarrow 4W$? $f_c = 22\text{ MHz}$

$4W'$
$4W''$

If $\dfrac{f_c + W}{4W}$ is an integer, then $f_s = 4W$ ✓

Else $\left\lfloor \dfrac{f_c+W}{4W} \right\rfloor = K$ then, $f_s = \dfrac{f_c + W}{K} = 4W' > 4W$



$4W = 8\text{ MHz}$

$\dfrac{24}{8}$ ✓ integer.

$\therefore \boxed{f_s = 8\text{ MHz}}$ ✓

$$s(t) = m_1(t)\cos 2\pi f_c t + m_2(t)\sin 2\pi f_c t$$

$$t \to kT_s = \frac{k}{4W}$$

$$f_c + W = L \cdot 4W$$

$$f_c = (4L-1)W \qquad L>0$$

$$s\left(\frac{k}{4W}\right) = m_1\left(\frac{k}{4W}\right)\cdot\cos\left(\cancel{2}\pi\cdot(4L-1)\cancel{W}\cdot\frac{k}{\cancel{4W}_{2}}\right) + m_2\left(\frac{k}{4W}\right)\sin\left(2\pi\,(4L-1)W\cdot\frac{k}{4W}\right)$$

$$(4L-1)k\,\frac{\pi}{2}$$

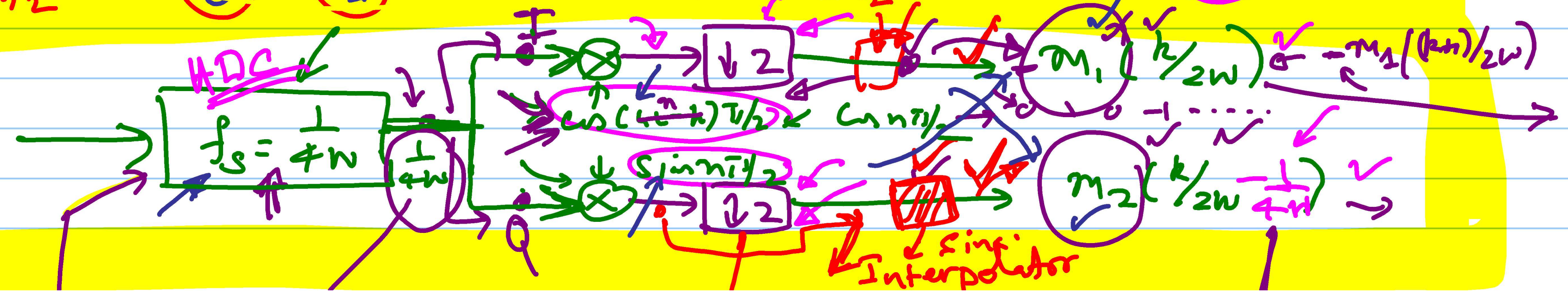
$$n = (4L-1)k$$

k'	1	2	3	4	5	6	7	8 .....
m_1	1	0	-1	0	1	0	-1	0
m_2	0	1	0	-1	0	1	0	-1

$\frac{1}{4W}$

Alignment

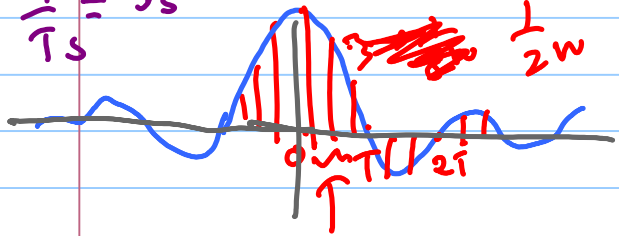
Timing Recovery

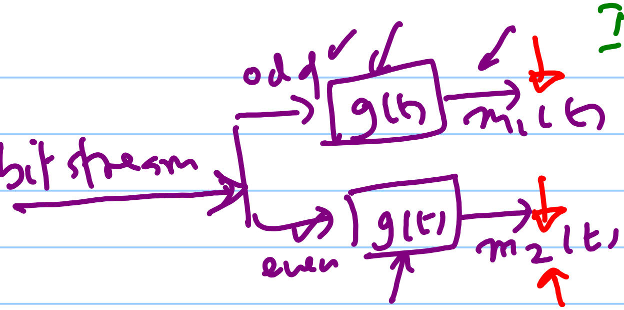


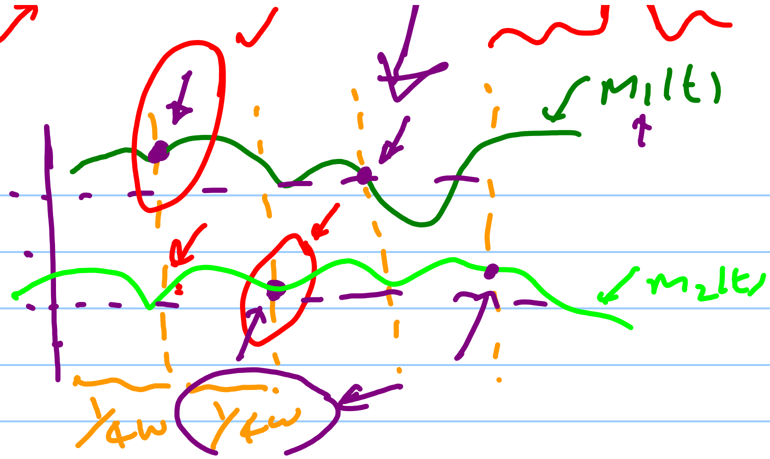
Lesson #2 (C4 -- Feb. 01, 2022)

$$f_c + W = L\cdot(4W)$$

$$\frac{1}{T_s} = f_s = 4W$$

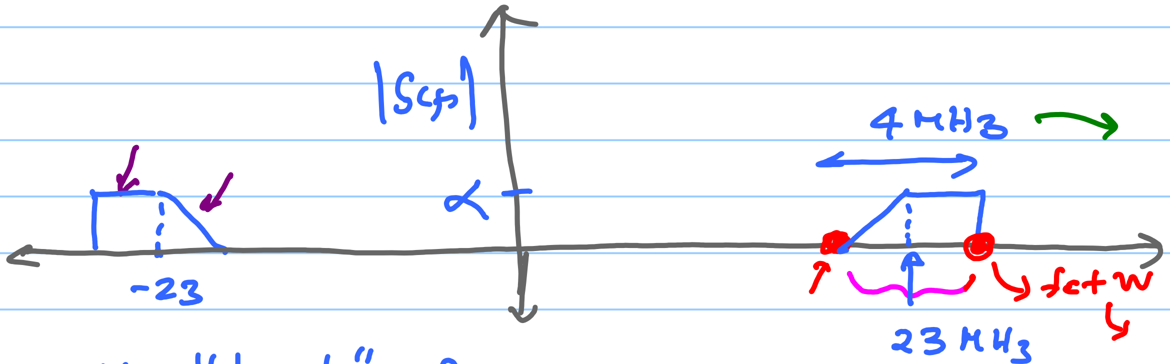




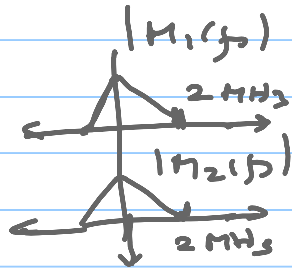


Example

$f_c = 23$ MHz



$\rightarrow$ $2W = 4$ MHz



a* Find the "least" f_s for this QCM signal

b* Plot the spectrum of the sampled QCM signal

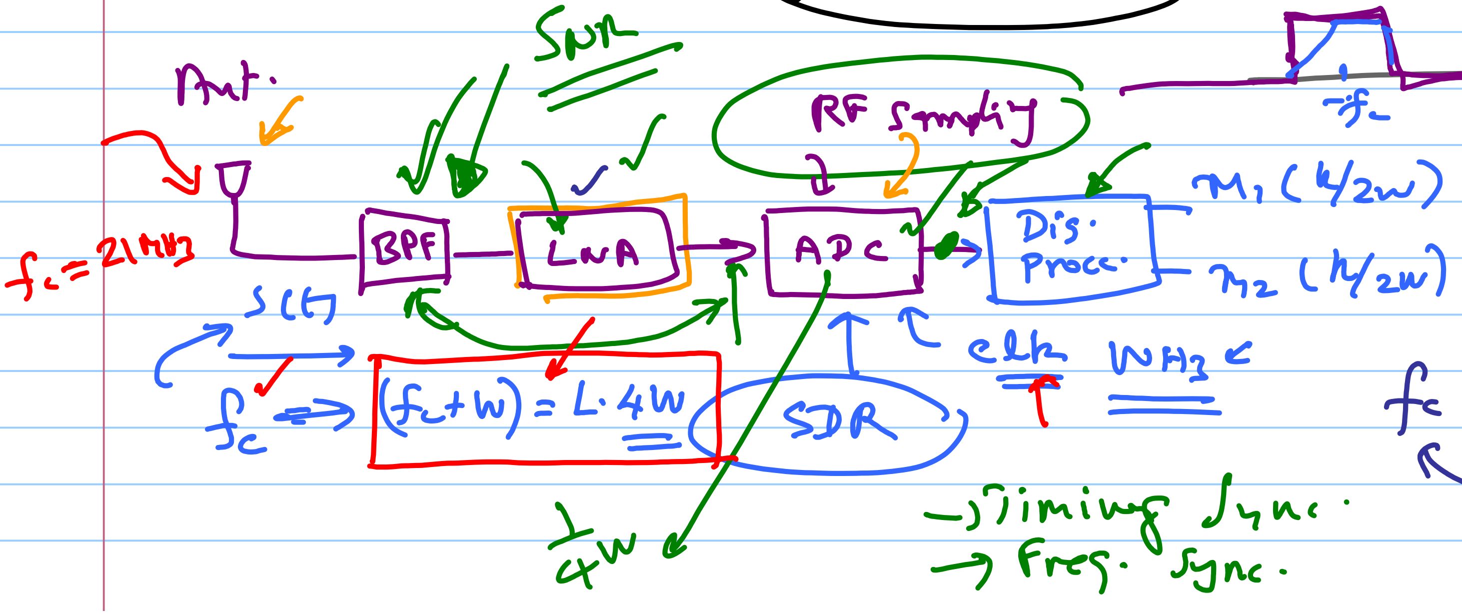
$f_s = \frac{25}{3}\,\text{MHz} \simeq 8\frac{1}{3}\,\text{MHz}$;

$f_c + W \leftrightarrow 4W$

$\left\lfloor \frac{f_c+W}{4W} \right\rfloor = \left\lfloor \frac{25}{8} \right\rfloor = 3$.

$\therefore\ f_s = \frac{25}{3} = \boxed{8\tfrac{1}{3}\,\text{MHz}} = 4W' > 4W$ (8 MHz)

* Repeat for $f_c = 21$ MHz ;



$e^{-j2\pi \Delta f_c \cdot k}$

→ Timing Sync.

→ Freq. Sync.

$f_c \rightarrow f_c'$

$f_c' - f_c = \Delta f_c$

CFO