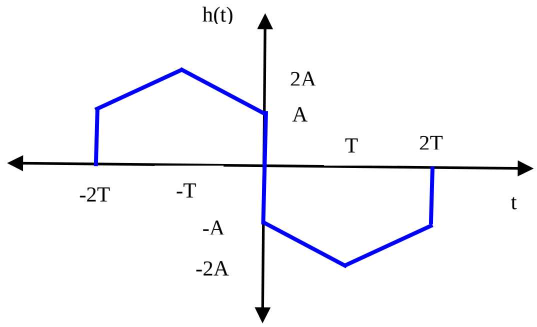
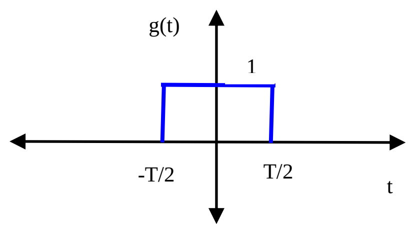
Department of Electrical Engineering
Indian Institute of Technology, Madras

EE 3005: Communication Systems

February 22, 2025 **Tutorial #2** KG/IITM

1. Express the Fourier transform H(f) of h(t) in terms of the Fourier transform G(f) when g(t) and h(t) are given as below:





2. A real, bandpass, QCM signal $s(t) = m_1(t)Cos(2\pi f_c t) + m_2(t)Sin(2\pi f_c t)$ with carrier frequency f_c=200KHz conveys two message signals $m_1(t)$ and $m_2(t)$ of one-sided bandwidth of W=5KHz each. Let $S(f)$, $M_1(f)$, and $M_2(f)$ denote the (complex) Fourier transforms of $s(t)$, $m_1(t)$ and $m_2(t)$, respectively. Given $M_1(f$=2KHz) = a + jb and $M_2(f$=2KHz) = c + jd where j = sqrt(-1), determine the following in terms of a, b, c, and d. The number inside the brackets can be assumed to be in KHz.
(a) $M_1(-2)$ and $M_2(-2)$
(b) $S(202)$ and $S(198)$
(c) $S(-198)$

3. Prove that the angle-modulated signal $s(t) = e^{-\hat{\varphi}(t)}Cos(2\pi f_C t + \varphi(t))$ has no frequency components in $-f_C < f < f_C$, where $\hat{\varphi}(t)$ is the Hilbert transform of $\varphi(t)$.
Hints for the proof: First show that this signal can be rewritten as $Re\{e^{j(2\pi f_C t + \varphi_+(t))}\}$ where $\varphi_+(t)$ is the pre-envelope signal. Then use the fact that for a complex quantity z, one can write $Re\{z\} = \frac{1}{2}(z + z^*)$ to get an alternate expression for $s(t)$. Then, use the series expansion $e^{j\varphi_+(t)} = \sum_{n=0}^{\infty} \frac{j^n}{n!} \varphi_+^n(t)$ and take Fourier transform on both sides. You should finally get the result by recalling the property about the FT of the pre-envelope signal.

4. The following questions from Chapter 3 of the text book (E-version), pp-143 onwards:

3.14 to **3.16,** and **3.17*.** ***** *these questions could have a higher order of difficulty to solve*

5. Reading exercise: Up to Section 4.1 in the textbook (up to page 159). Attempt question 4.18.

6. Find the compact ortho-normal basis set, and using it, make a clear labeled plot of the signal constellation for the signal set shown in Fig-1.

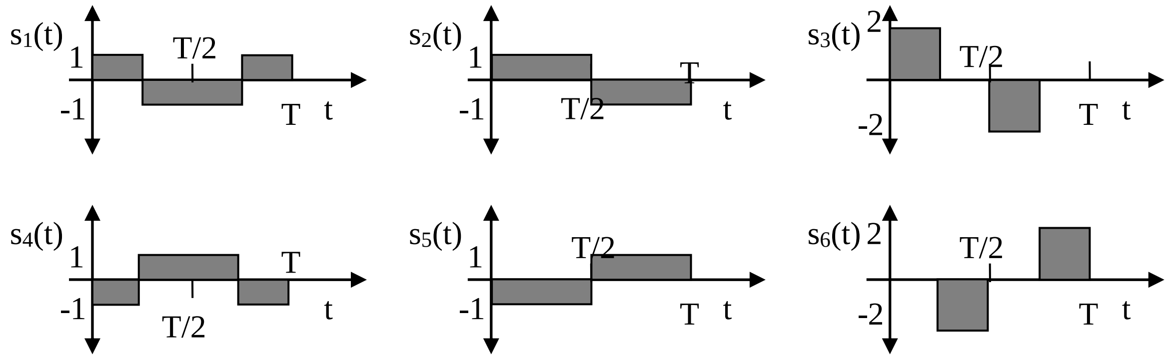


Fig-1

7. The signal g(t) is sent through a channel with impulse response h(t), where the two functions are shown in Fig-2. Make a labeled plot of the ideal matched filter's impulse response. *Hint:* Assume single-shot communication.

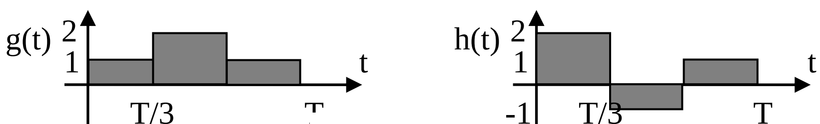


Fig-2

8. Find the autocorrelation function S(t) for the given signal g(t) in Fig3. Draw the signal S(t).

where

$$S(t) = \int_{-\infty}^{\infty} g(\tau)g(t-\tau)d\tau$$

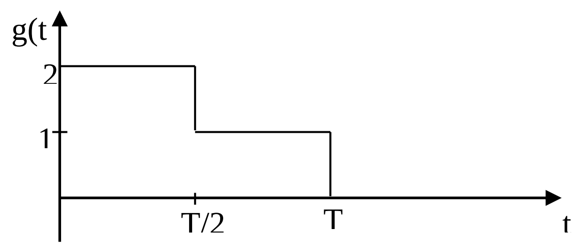
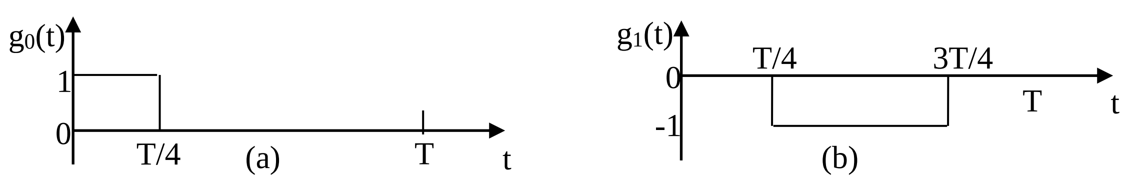


Fig-3

9. Find out the compact basis function for signals given in Fig-4.



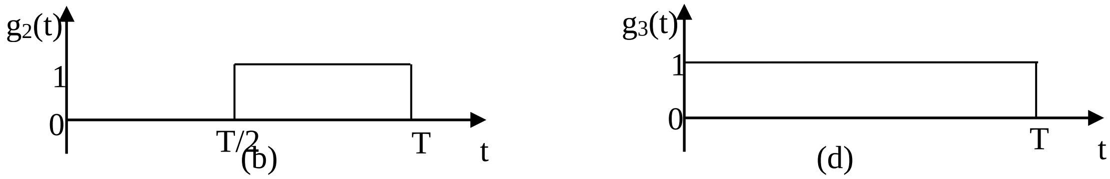


Fig-4

10. Consider the signal set shown in Fig-5 below.

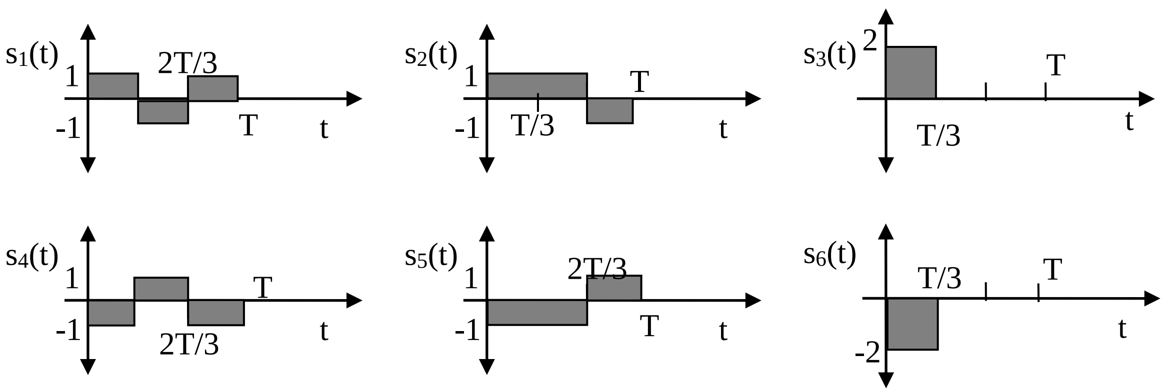


Fig-5

(a) Find a compact orthonormal basis set for this signal set. Sketch these functions.

(b) Using this, make a clear labeled plot of the corresponding signal constellation.

(c) In terms of the average energy E_a of the constellation, what is the minimum distance (i.e., 2d) of the signal set?

11. Find the minimum distance and multiplicity for given signals in Fig-6.

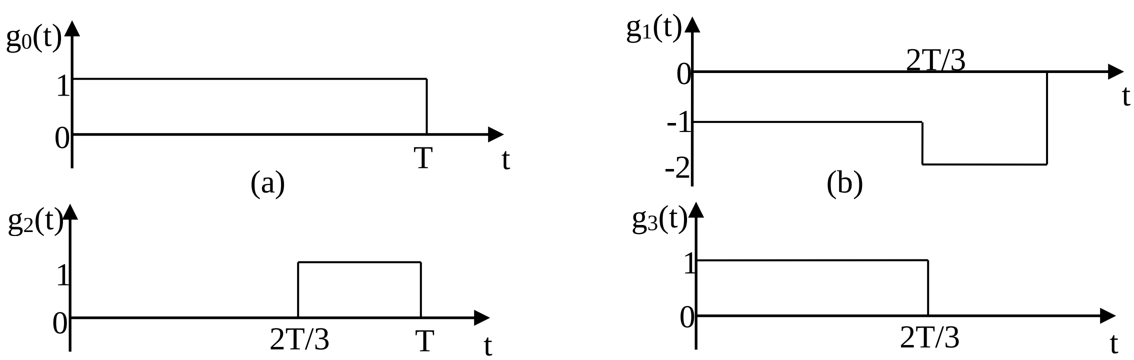


Fig-6