EE-1100 Basics of EE @ IIT Palakkad

Oct. 23, 2016 **Tutorial #4** KG / IITM

Traffic Engineering – Erlang B formula, Multistage Switching

1. What is the amount of traffic E that can be accepted by M=2 servers if a high blocking probability P_b = 0.50 is allowed?
(a) Repeat when the allowed P_b = 0.02.
(b) Defining the output utilization factor $\gamma = (1- P_b)E / M$, what is it for the above 2 cases of P_b?

2. Repeat the steps in Pbm. 1 for the case of M=3 servers.

3. Given a population of N=20,000 users, each offering Eu=0.04 Erlangs of traffic, define a 3-stage blocking switch with k sub-arrays in the middle-stage, each containing 250x250 cross-points such that the blocking probability $P_b \leq 10^{-3}$. Use the Lee graph approach to find this least value of k.
(a) Determine the number of cross-points for the above switch.
(b) Rework value of k and part (a) if we require $P_b \leq 10^{-6}$.
(c) For the same size of the middle-stage sub-arrays (i.e., same size of m and n) as in (a), define a non-blocking switch. How does the complexity of this switch compare to (a)?
(d) For these N=20,000 users, what will be the least complexity of a 3-stage non-blocking switch if one had the flexibility to choose any n (and k) ? (Recall in our notation: N=nm)

4. A total of N=4096 lines have to be switched, where each line offers Eu=0.05 Erlangs of traffic. All the 3 stages of the switch are to be built using sub-arrays of size 64x64 (where in the input and output stages, not all lines need be utilized if k <64).
(a) Define a blocking switch such that blocking probability $P_b \leq 10^{-3}$. What is it's complexity (including the unutilized cross-points)?
(b) Is it possible to build a non-blocking 3-stage switch in this case? If so, specify the same and it's complexity. (*Hint*: In a non-blocking 3-stage switch, imagine the ith user calling the jth user and they belong to different nxk blocks. For the worst case scenario, the other n-1 users from the input block where the ith user belongs could be busy, and similarly, the other n-1 users where the jth user belongs could be busy in the output block.)

5. The first 400 inlets carry users with Eu=0.05 Erlangs while the next 600 inlets carry users with Eu=0.01 Erlangs. Given that the users are grouped into blocks of n=50 each, define a 3-stage blocking switch with overall $P_b \leq 10^{-2}$. What is the total number of cross-points in this switch? *Hint*: The overall P_b is computed by considering the 4 cases, namely user from set1 calls another user in set1, or user from set1 calls user from set2, etc.

6. Consider a population of N=4000 users, each of E_u=0.01 Erlangs. Design a 3-stage blocking switch of least complexity such that the blocking probability P_b=10^{-4} or less. What is k, and the total number of cross-points for this switch? *Hint:* To minimize the total number of cross-points, choose the input sub-array dimension n "appropriately" where $N/m=n$.

7. (Optional) Consider the 5-stage switch in the book, first described in page 237, Fig. 5.9. Here, blocking is introduced also in the middle stage(s). The input has N/n_1 sub-arrays, each of dimension n_1x k_1, where N is the total population to be served by this switch. The middle-stage (which is actually a blocking switch with 3-stages) has k_1 sub-arrays, each of size N/n_1x N/n_1. Each of these sub-arrays has $N/(n_1$x $n_2)$ sub-arrays, of dimension n_2xk_2 where k_2 is the number of middle stage sub-arrays (each of dimension $N/(n_1$x $n_2)$ x $N/(n_1$x $n_2)$). Assume each user offers E_u Erlangs of traffic.

(a) Prove using the Lee-Graph approach that blocking probability of the 5-stage switch is given by $P_b = \left\{1 - q_1^2\left[1-(1-q_2^2)^{k_2}\right]\right\}^{k_1}$ where $q_1 = (1-p_1)$ with $p_1 = \frac{n_1 E_u}{k_1}$ and $q_2 = (1-p_2)$ with $p_2 = \frac{n_2 p_1}{k_2}$.

(b) For N=50,000, and n_1=50 and n_2=50, find the 5-stage switch with minimum number of cross-points so that P_b=10^{-8} or less. Assume E_u = 0.01 Erlangs each.

(c) Can you find a better choice of n_1 and n_2 for this case? (i.e., a choice that will minimize the number of cross-points further?)