1. [1.5+1.5+2 = 5 marks] The voltage across a load Z is given by $v(t) = 20\cos(\omega t - 30^\circ)$ V, and the corresponding current through it is measured to be $i(t) = 2\sin(\omega t + 30^\circ)$ A. Find the following:

(a) Complex power (b) Apparent power, and the (c) Load impedance Z.

$V = \frac{20}{\sqrt{2}} \angle -30^\circ = 10\sqrt{2}\angle -30^\circ$ $\quad I = \frac{2}{\sqrt{2}} \angle -60^\circ = \sqrt{2}\angle -60^\circ$

$\frac{V}{I} = 10\angle 30^\circ = 1.542 - 9.881 = Z$

$V_{rms} I_{rms} \cos\phi = 20\frac{\sqrt{3}}{2} = 10\sqrt{3}$

$V_{rms} I_{rms} \sin\phi = 20\times\frac{1}{2} = 10$

$20 + j10 = S$ $\quad |S| = 10\sqrt{5}$

a) Complex power $\longrightarrow$ $10\sqrt{3} + j10$

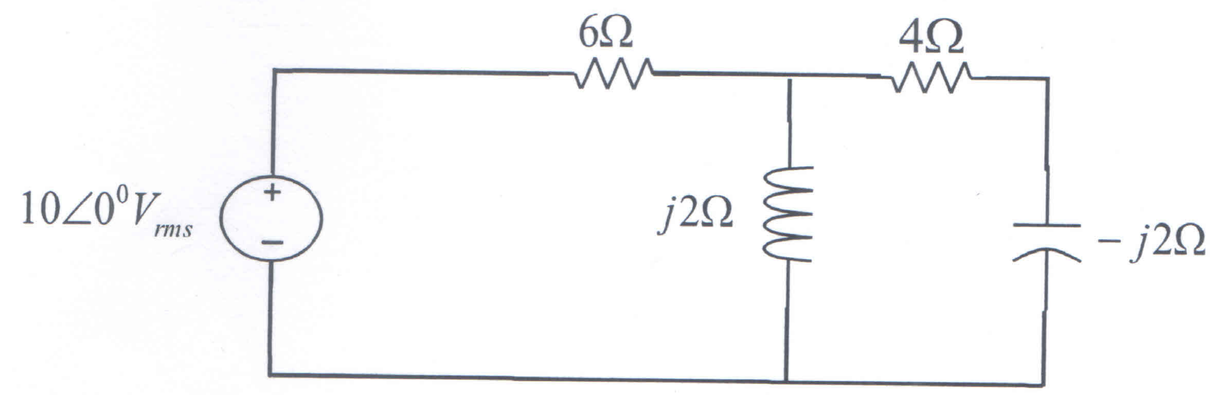
b) Apparent power $\longrightarrow$ $10\sqrt{5}$ 20

c) Load impedance Z $\longrightarrow$ $1.542 - 9.881$

$5\sqrt{3} + 5j$

B

2. [2.5+2.5 = 5 marks] In the circuit below, calculate the following:
(a) Power factor seen by the source
(b) Average power supplied by the source.



$Z_{eq} = 6 + \dfrac{j2 \times (4-j2)}{j2 + (4-j2)} = 7 + 2j$

$= \sqrt{53}\ \angle 15.95$

$Pf = 0.9615$

$I_{rms} = \dfrac{10\angle 0}{\sqrt{53}\ \angle 0.2747\ \angle 15.95} = 1.373\ \angle -15.95$

avg power $= 1.373 \times 10 \times 0.9615$

$= 13.20$ watt.

3. [3+3 = 6 marks] In a small town with "talkative" users, each of them offer Eu=0.05 Erlangs of voice traffic. Use the Erlang-B formula to answer the following questions:

(a) Kumar commissions a trunk exchange with K=2 trunk lines (servers). If he plans to allow up to 10% blocking (i..e, P(B)=0.1), how many users can he serve in the town? Express your answer after rounding to the nearest integer number of users, N.

(b) Instead, his competitor Kumaran commissions a trunk exchange with K=4 trunk lines. With this, Kumaran claims that he can serve with a *lower* blocking percentage, twice the number of users that Kumar serves (i.e., 2N users). Is Kumaran correct? Find this percentage and comment.

(a) $$P(B) = \frac{E^K/K!}{\sum_{l=0}^{K} E^l/l!}$$

For $K=2$ and $P(B) = 10\% \Rightarrow 0.1$,

$$0.1 = \frac{E^2/2}{1 + E + E^2/2}$$

Solving, $E = \frac{1+\sqrt{19}}{9} = \boxed{0.5954}$; 3'

with $Eu = 0.05$, $N = \frac{E}{Eu} = 11.91 \approx \boxed{12 \text{ users}}$;

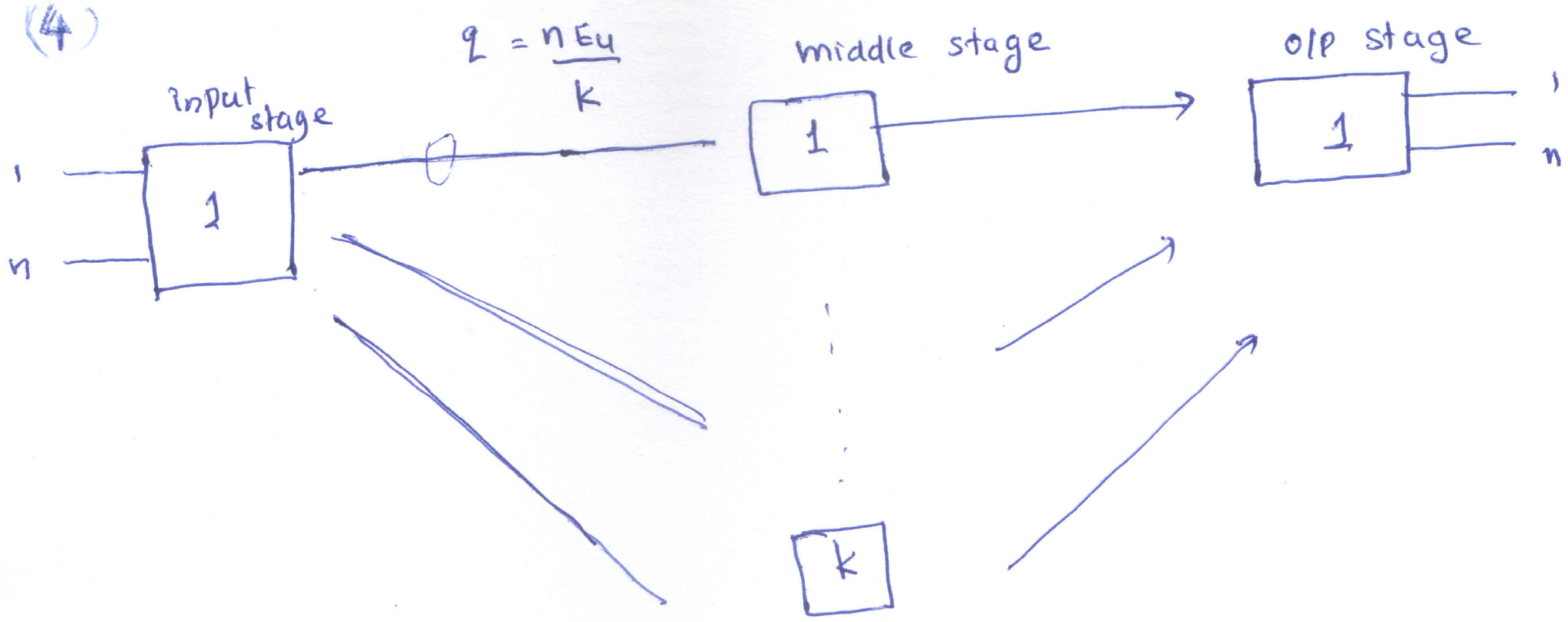
$\therefore$ Kumar serves $N=12$ users

(b) For kumaran, 2N users $\Rightarrow 2 \times 12 = 24$ users will bring in $24 \times 0.05 = 1.2$ Erlangs of traffic. With $K=4$ servers, the P(B) will now be (for kumaran's switch) 3'

$$P(B) = \frac{1.2^4/4!}{1 + \frac{1.2}{1} + \frac{1.2^2}{2} + \frac{1.2^3}{6} + \frac{1.2^4}{24}} = 0.026226$$

$\Rightarrow \boxed{2.62\%}$ (much lower than 10%)

(4)



Given $N = 1,20,000$ $E_u = .01$ $P(B) = .005$

(a) $n = 200$ $q = \frac{200 \times .01}{k} = \frac{2}{k}$

$$P(B) = \left(\frac{q}{k} - \left(\frac{1}{k}\right)^2\right)^k < .005$$

solving $\Rightarrow$ $k = 8$ (2 marks).

(b) $n = 400$ $q = \frac{400 \times .01}{k} = \frac{4}{k}$

$$P(B) = \left(\frac{8}{k} - \left(\frac{4}{k}\right)\right)^k < .005 \Rightarrow k = 11 \quad (2 \text{ marks})$$

(c) $n = 600$ $\quad q = \frac{6}{k}$

$$P(B) = \left(\frac{12}{k} - \left(\frac{6}{k}\right)^2\right)^k < .005 \quad \Rightarrow \underline{\underline{k = 14}} \qquad (2 \text{ marks})$$

(d) Based on above 3 values of k for $n = 200$, 400, & 600 we can compute the 3 stage switch complexity using $C = 2Nk + k\left(\frac{N}{n}\right)^2$

$n = 200$	$n = 400$	$n = 600$
48 000 00	3630 000	3920000
	↑	
	Least complex. (2 marks)	

Comment: This answer is not independent of N.

For Small N, $n = 200$ will be best ~~f~~ for large N, n must