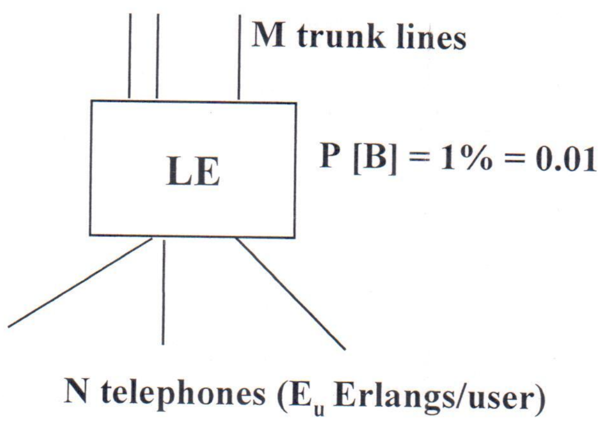
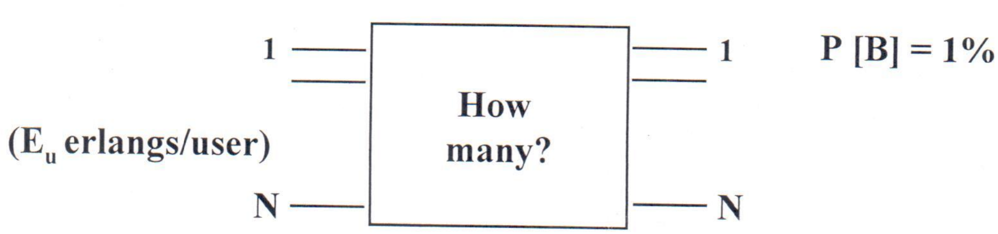
Circuit Switched Telephony

Can pose two interesting problems to address the complexity required at the switch

Question 1 : Given N users (subscribers), each of E_u Erlangs, how many trunk lines M are required for an LE with 1 % blocking?



Question 2 : For a N x N LE {actually (N + M) x (N + M) with M trunk lines}, how many cross-points are required for a 1% blocking switch?



Note : An N x N non-blocking switch requires $N.(N-1) \approx N^2$ cross points

What is Erlang Capacity?

- Erlang measures the amount of usage (or busyness) of a user or a set of users

  eg: If a user uses his/her telephone (on the average) of 15 mins every 1 hour, then he/she is a $\frac{15}{60}$ = 0.25 Erlang user

- Typically, voice users generate 0.05 to 0.1 Erlangs (and data users on the telephone line have $E_u = 0.2$)

- E_u = Avg. call arrival rate x Avg. call holding time

- <u>Question 1 Re-posed</u> : Given M servers, how much Erlang Capacity (E) or traffic handling capacity does it offer for 1% P[B]?

- Once E is found, then E/E_u will be the number of subscribers M servers can support at 1% blocking probability.

Erlang B Formula

(or, "blocked calls cleared" model)

- Serving without queuing model. With M servers, on the average E users can be seved *every* moment with 1% P[B]

  $\Rightarrow$ offered traffic by M servers = E erlangs

- Erlang B formula

$$P[B] = \frac{(E)^M / M!}{\sum_{k=0}^{M} (E)^k / k!}$$

- Example : Given M=2 and if we require P[B] = 0.01(1% Blocking) what is E?

$$0.01 = \frac{(E)^2 / 2!}{\sum_{k=0}^{2} (E)^k / k!} = \frac{E^2/2}{1+E+E^2/2}$$

$$\text{i.e.,} \quad \frac{E^2}{2}(1-0.01) - 0.01E - 0.01 = 0$$

$$\Rightarrow \quad E = 0.1526 \text{ (taking positive root)}$$

- Therefore, with M = 2, we can support $N = \frac{E}{E_u}$ users. If E_u = 0.05, then we can support 0.1526/0.05 $\approx$ 3 users

- Exercise : with P[B] = 50% show that we obtain E = 2.732 for M = 2 servers ($\Rightarrow$ we can support 54 users !)

- For small number of users, the Erlang B formula above is not accurate; it only yields an *upper bound* on P[B]

Application of Erlang B Formula

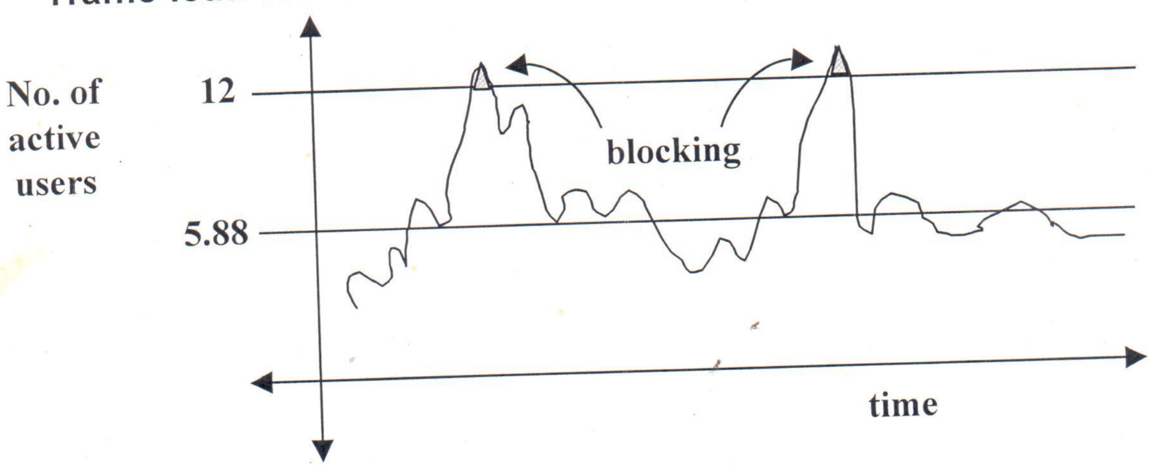
- For same number of servers, the offered traffic (capacity) can be increased only at the cost of increasing P[B]

  eg: M=12; assume E_u=0.1 Erlangs; calculating from tables we have:

  1 % is typical for telecom

P[B] %	0.01	0.1	0.5	1.0	10	20	50
E	3.21	4.23	5.28	5.88	9.47	12	22. 2
N=E/Eu	32			59			222

- Note : E $\rightarrow$ is offered traffic (Erlang Capacity). The actual maximum carried traffic at any time = M = 12 Erlangs
- Traffic-load versus time description



Blocking will be < 1% provided the total erlangs generated by the N users does not exceed 5.88

Trunking Efficiency

- As M increases, Erlang Capacity E increases
  (offered traffic)
- Also, as M increases, Erlang Capacity per server also increases

eg: P[B] = 0.5%

	M	12	24	48	96
	E	5.28	14.2	34.2	77.2
Erlang/server →	E/M	0.44	0.59	0.71	0.80

$\Rightarrow$ Co-locate servers in order to utilize them better!

- For circuit switching, when real-time communication is essential, "blocked calls cleared" model is more appropriate
- Note that Erlang B model is a "blocked calls cleared" model. Is it better to queue the blocked users?
- *This yields a "blocked calls delayed" model*
- P [delay > 0] (and not P[B]) is the Qos for this model
- Interestingly, the corresponding Erlang C formula offers fewer erlangs for same M !!

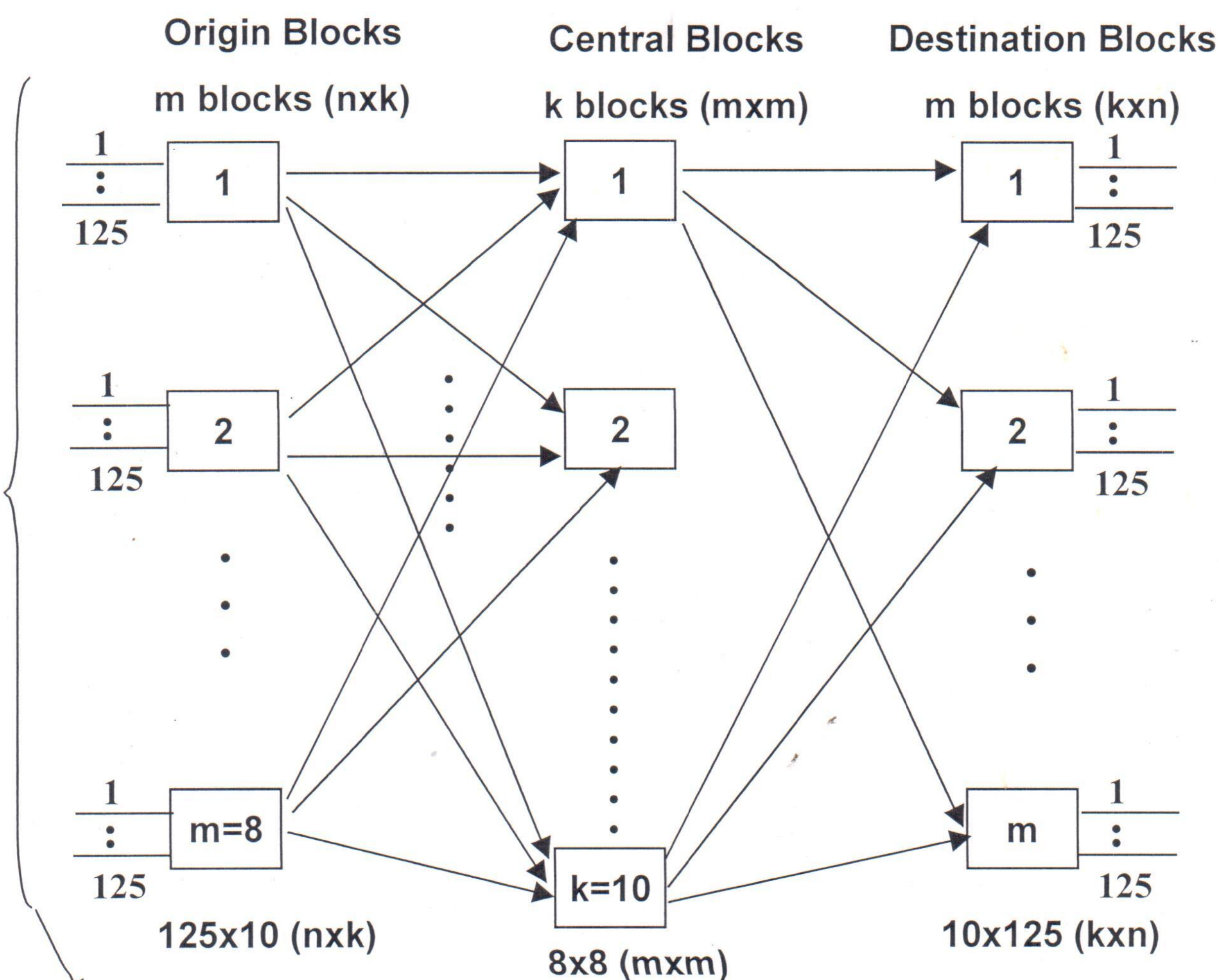
Returning to Question 2,

Given N users, how many cross points are required by a (multistage) switch if 1% Blocking Probability is ok?

<u>Recall:</u> Non-blocking switch requires N . (N-1) ≈ N^2 cross points

Multi-Stage Switching

eg: N=1000 ; break into blocks of n = 125 such that m = N/n = 8;



Total
N = m . n
= 8x12
= 1000 subscribers

Multistage Switch Complexity

- m . (n x k) + k . (m x m) + m . (k x n)

  origin block — central block — destination block

  ↑ blocking — ↑ non-blocking — ↑ blocking

  $= 2\,(\underbrace{m\,.\,n}_{N})\,k + km^2$

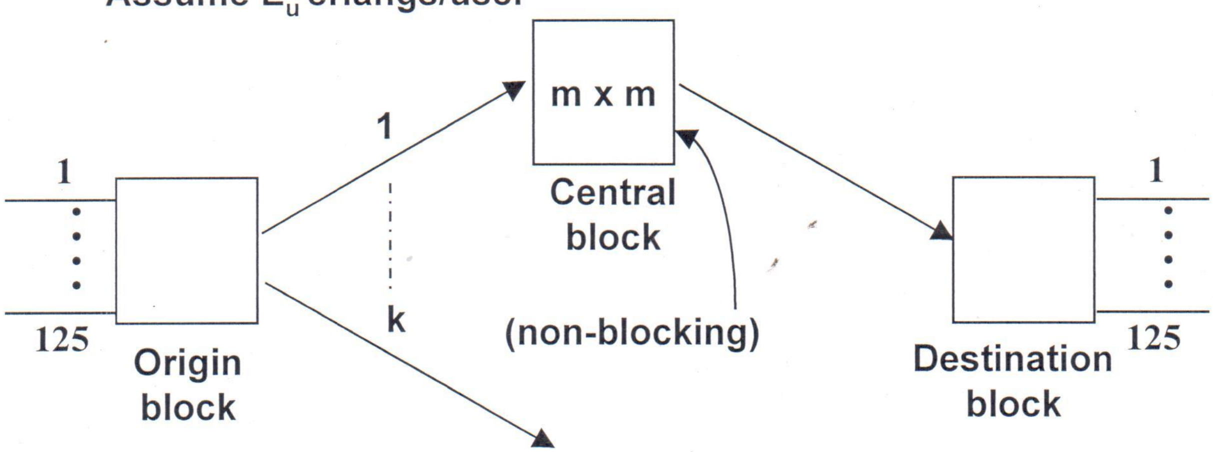
  $= k\,.\left[2N + \left(\frac{N}{n}\right)^2\right]$

- eg: With N=1000, n=125, k=10

  ⇒ 10 . [2000 + 8^2] = 20,640 cross points (instead of 1000 x 1000 = 10^6 for non-blocking switch !!)

- What is the P[B] for this blocking switch?

  Assume E_u erlangs/user



With n = 125, generated traffic for the block = 125 E_u

This gets divided (equally) between the k = 10 lines

Multistage Switch P[B]

- Therefore traffic on any <u>one</u> line at the output of the origin block = $\frac{125.Eu}{10}$ = q

  Eg. If $E_u = 0.05$, $\Rightarrow$ q = 0.625
- Therefore, the probability that any one line from origin block to central block is occupied = q

  $\Rightarrow$ Probability that it is free = 1-q
- Strictly speaking, the factor q is only proportional to the actual probability; observe that q is well defined only when the ratio $n\,E_u/k$ is less than 1
- Similarly, the probability that one line from the destination block to a central block is free also equals 1-q
- But, probability that two such lines to the <u>same</u> central block are free (only then can a connection be established) = $(1-q)^2$
- Therefore, the probability that any one of the k central blocks is not available for a call establishment = $1 - (1-q)^2$

  $\Rightarrow$ P[call cannot go through any the k central blocks] = $(1-(1-q)^2)^k$
- In other words, blocking probability of this three stage switch is $P[B] = (1-(1-q)^2)^k$

Blocking Probability P[B]

- To summarize

$$P[B] = \left[1-(1-q)^2\right]^k$$

where $q = \frac{n.E_u}{k}$ and $n = \frac{N}{m}$;

- Example : q = 0.625 ⇒ P[B] ≈ 0.22
  (k = 10) = 22%
- Example : k = 20 ⇒ q = 0.3125
  ⇒ P[B] = 0.0276 (0.000276%)
- Example : k = 13 ⇒ P[B] = 1.68%
  k = 14 ⇒ P[B] = 0.596%
- Extend the multi-stage switch idea to a group of m LEs connected to a TE with k lines each