Department of Electrical Engineering

EC-3201 : Communication Systems

March 2012 **Tutorial #2** KG / IITM

Note: The problems must be worked out, with all important calculations clearly shown in order to be eligible for full credit. Your answer sheet must be turned in on or before the Thursday class at 11am, on March 22, 2012. Negative marks will be awarded if copied answers are turned in. Each question carriers 5 marks, unless otherwise specified.

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1. A QCM signal $s(t) = m_1(t)Cos(2\pi f_c t) + m_2(t)Sin(2\pi f_c t)$ has the two message signals $m_1(t)$ and $m_2(t)$ of one-sided bandwidth of W_1=3KHz and W_2=4KHz, respectively, and f_c=30KHz.
(a) Find the minimum band-pass sampling rate $f_s=1/T_s$ that will give un-aliased samples of the two signals.
(b) Assuming that the spectrum of $m_1(t)$ has a "triangular" shape between -3KHz to +3KHz, make a labeled, rough sketch of the spectrum of the samples $m_1(kT_s)$ between -40KHz and +40KHz.

2. For each of the below signals, which have a peak-to-peak swing between -1volts and +1volts, make a rough sketch of the modulated FM and PM signals, bringing out the salient aspects of each clearly. Assume that the carrier frequency f_c=1KHz, and that the time-duration T in the plots below is T=1sec.

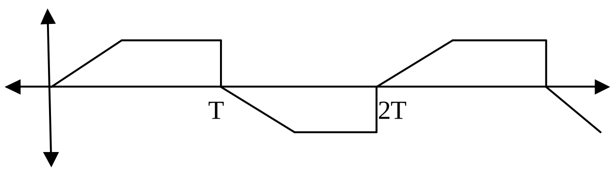


Fig. 2a

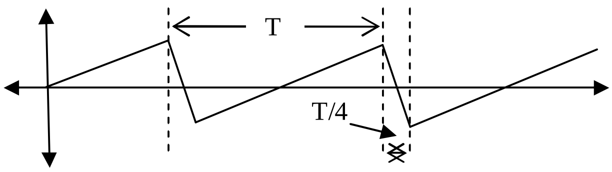


Fig. 2b

3. A 10ppm clock error exists in the incoming signal to the receiver, which in turn has a 5ppm clock error. If the nominal bit-rate is 1Mbps, how much time elapses between (bit-level) slips? If this slip rate must not be greater than 10^{-3} slips/sec, what must be the new spec. on the total clock error?

4. In a system using elastic buffer with 2-frame memory, one frame slip occurs every 36 minutes in a system with a frame rate 150 frames/sec.

a) What is the ppm (parts per million) of the crystal used in the receiver?

b) Now, in order to have only one (frame) slip every 36 hours, what should the ppm be?

5. A digital multiplexer has 16 incoming streams, each with 2.048MBps rate and clocks with ±10ppm. The multiplexer adds a 32-bit framer (unique word) and a 16-bit CRC, and then stuff-bits (and the stuff-bit length indicators) are inserted to assemble a frame every 1sec.

(a) Assuming that this operation is done so that each of the 16-streams are multiplexed bit-by-bit, what will be the multiplexed bit-rate? Make a rough sketch of the frame indicating the various fields.

(b) If the frame duration is reduced to 10msec, but with the framing and CRC overheads remaining the same, what will be the output bit-rate? Comment with reference to your answer in (a).
(c) Supposing elastic buffers with byte-level memory is used (byte is 8-bits), and the 16-streams are multiplexed byte-by-byte, what will be the multiplexed bit-rate for the case when frame duration is 1sec? Again, compare your answer with that in (a) and comment.

6. In a intermediate level digital multiplexer, 4 input streams arrive with rates and clock ppm given as follows: 5Mbps (1ppm), 10Mbps (1ppm), 10Mbps (2ppm), and 5Mbps (5ppm). If a 32-bit frame header and a 32-bit CRC are added to every 100msec frame assembled by this multiplexer along with appropriate stuff-bits (and indicators), answer the following:
(a) Describe a simple bit-by-bit multiplexing strategy.
(b) Make a rough sketch of the assembled frame, indicating the various important fields.
(c) What is the output bit-rate?

7. [6x5 = 30 marks] Referring to the Tutorial #2 of year 2008 (also enclosed), answer the following 6 questions:

- ➔ (Pbm. 5a to 5d) Switching: Problems 8,9,10, and 11
- ➔ (Pbm. 5e and 5f) Erlang-B formula application: Problems 13 and 14

8. For the switch considered in Problem 8 (in tutorial #2 of year 2008), use the blocking probability expression following the work of Jacobaeus (which does not assume that the paths from input-to-middle stage and paths from output-to-middle stage are independent) given in eqn. (5.10) in page 239 of Bellamy "Digital Telephony". What will be the new value of *k* for this case?

9. Consider a population of N=4000 users, each of E_u=0.01 Erlangs. Design a 3-stage blocking switch of least complexity such that the blocking probability $P_b=10^{-4}$ or less. What is *k*, and the total number of cross-points for this switch? *Hint:* To minimize the total number of cross-points, choose the input sub-array dimension *n* "appropriately" where $N/m=n$.

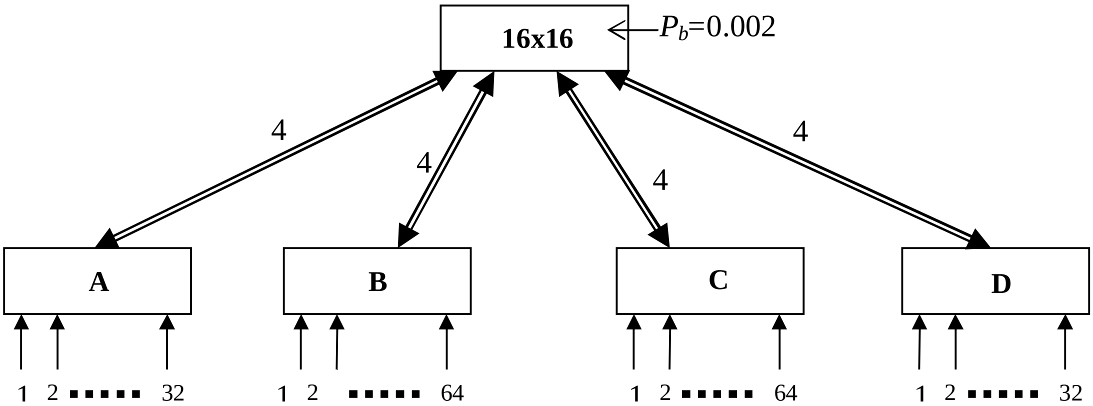
10. Consider the 5-stage switch in Bellamy's book, first described in page 237, Fig. 5.9. Here, blocking is introduced also in the middle stage(s). The input has N/n_1 sub-arrays, each of dimension n_1x k_1, where N is the total population to be served by this switch. The middle-stage (which is actually a blocking switch with 3-stages) has k_1 sub-arrays, each of size (N/n_1) x (N/n_1). Each of these sub-arrays has $N/(n_1$x $n_2)$ sub-arrays, of dimension n_2xk_2 where k_2 is the number of middle stage sub-arrays, each of dimension $N/(n_1$x$n_2)$ x $N/(n_1$x$n_2)$. Assume each user offers E_u Erlangs of traffic.

(a) Prove using the Lee-Graph approach that blocking probability of the 5-stage switch is given by
$P_b = \left\{1 - q_1^2\left[1-(1-q_2^2)^{k_2}\right]\right\}^{k_1}$ where $q_1 = (1-p_1)$ with $p_1 = \frac{n_1 E_u}{k_1}$ and $q_2 = (1-p_2)$ with $p_2 = \frac{n_2 p_1}{k_2}$.
(b) For N=50,000, and n_1=50 and n_2=50, find the 5-stage switch with minimum number of cross-points so that $P_b=10^{-8}$ or less. Assume E_u = 0.01 Erlangs each.

(c) Can you find a better choice of n_1 and n_2 for this case? (i.e., a choice that will minimize the number of cross-points further?)

11. Consider the design of a 3-stage blocking switch for N=10,000 users, each offering E_u=0.02 Erlangs. All the sub-arrays in the switch will be of dimension 100×100 (and in the input and output stage sub-arrays, not all these links need to be used). Determine the (least) number of middle-stage subarrays, k, such that the blocking probability P_b is 10^{-6} or lower. What is the total number of 100×100 sub-arrays used in the switch?

12. We have 4 sub-switches A,B,C,&D, where the trunk traffic is served by 4 trunk lines each, as shown in the figure below. The trunk lines are in turn switched by a 16x16 blocking switch with Pb=0.002. Assume that each of the trunk inlet ports (to A,B,C,&D) offer E_u=1/16 Erlangs of traffic, and only the trunk-calls (i.e., calls to #s outside the sub-switch) are represented by the inlets.



(a) Find the blocking probabilities (for connecting to the trunk servers) at sub-switch A and also at sub-switch B. (*Hint*: use Erlang-B formula)
(b) What is the probability that a trunk user from switch A <u>can</u> connect to a trunk user in switch D?
(c) What is the probability that a trunk user from switch A <u>cannot</u> connect to a user in switch C?
(d) What is the <u>overall</u> average blocking probability of this switching trunk system?

13. From "Digital Telephony" J.C.Bellamy, 3rd Ed., (a) **<u>Reading</u>** from Chapter 5: Switching; pp. 225 to 261. (b) **<u>Solving</u>** from Ch-5, pp.274,: **5.2***, **5.3*** (Lee Graph only), **5.4*** thro **5.8***. None of these problems marked with "*" need to be turned in.