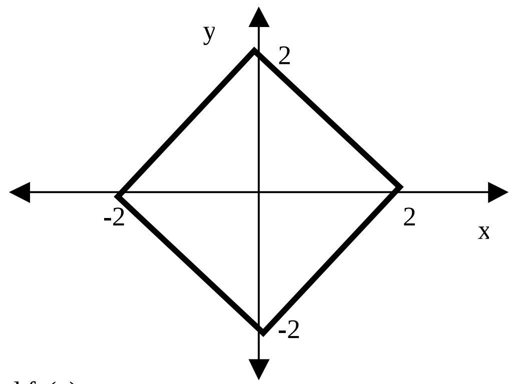
EC 305 : Communication Systems

Tutorial 1 : Sampling, Quantisation, & Line Coding

1. X is a (random) variable with an uniform probability density function (pdf) beween the two limits (a,b) specified simply as follows: $f_X(x) = 1/(b-a)$, if $a \le x \le b$; else, $f_X(x) = 0$. The mean value of X is defined by $m_X = E[X] = \int_a^b x\, f_X(x)\, dx$ and the power or variance is defined by $\sigma_X^2 = E[(X- m_X)^2]$. Using this, show that the variance of the uniformly distributed X is given by $\sigma_X^2 = (b-a)^2/12$.

2. Two random variables X and Y are uniformly distributed on the square shown below:



(a) Find and sketch $f_X(x)$ and $f_Y(y)$.
(b) If both of them are individually quantised using 4-level uniform quantisers, specify their Peq.
(c) What is the resulting number of bits per (X,Y) pair?
(d) Make a neat sketch of the quantization levels in both case (a) and case (b).

3. Optional Question (for the adventurer): For the above joinf pdf, can you construct a non-uniform vector quantiser? Again, this will be based on the Llyod-Max rule, with the difference being that the decision regions are defined by perpendicular bisectors to the line connecting the quantised values. What will be the number of bits/symbol required in this case?

4. A WSS random process has an acf given by $R_X(\tau) = \frac{A^2}{2} e^{-|\tau|} Cos(2\pi f_o \tau)$. Assume that the random process never exceeds 6 in magnitude, and that A=6.
(a) How many uniform quantisation levels are required to provide an SQNR of at least 40dB ?
(b) If we want to increase the minimum SQNR to 60dB, how should the required number of quantisation levels change?
(c) If f_o=1MHz, what is the bit rate you will require to send the quantised samples in both of the above cases?

5. The psd of a WSS process X(t) is given by

$$S_X(f) = \begin{cases} \dfrac{f + 5000}{5000}, & -5000 \le f \le 0 \\ \dfrac{-f + 5000}{5000}, & 0 < f \le 5000 \\ 0, & \text{otherwise} \end{cases}$$

and the maximum amplitude of this process is 6.

(a) What is the power content of this process?
(b) If this process is sampled at fs to guarantee a guard band of 2000Hz, then what is fs?
(c) At this sampling rate, if we use a linear PCM system with 256 levels, what is the resulting SQNR in dB?
(d) What is the resulting bit rate?
(e) If we need to increase the SQNR by atleast 25dB, how many levels are required? What is the new bit rate?

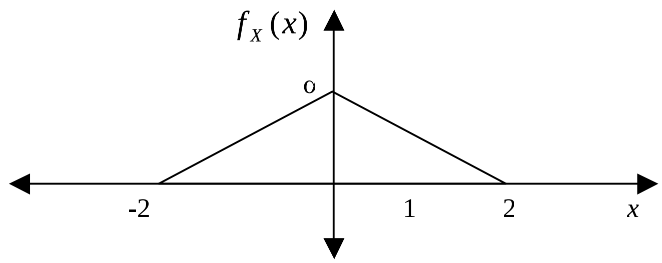
6. A source X with a triangular pdf as below is to be quantised to 4 levels:

(a) Find α.

(b) Assuming a simple uniform quantiser (the obvious choice being a_1=-1, Δ=1), find the pdf of the quantisation error given by $E=X-X_q$, where X_q is the quantised value.

(c) What is the SQNR for the uniform 4 level quantiser?

(d) Perform 3 iterations for the Llyod-Max non-uniform quantiser, starting with the initial decision regions as defined in part (b). What is the new SQNR?



7. Given a 10MHz signal g(t), with power 2watts, it is required to define a L-bit ADC such that signal to (quantization) noise ratio is at least 48 dB. What is the (least) bit-rate required to represent the sampled stream $g(kT_s)$? Assume power is the square of the (peak) voltage.

8. Reading: "Digital Telephony 3rd Ed." by J.C.Bellamy – Sec. **3.1** thro Sec.**3.9** (Skip details in Fig 3.34 and Fig 3.36).

9. Problems from "Digital Telephony 3rd Ed." by J.C.Bellamy, Chapter 3 (pp.158-160): **3.2, 3.3, 3.4, 3.5, 3.6, 3.14, 3.18, and 3.20*.**

10. Consider the following 12 bit sequence: 111010001011 to be sent using rectangular waveforms.

(a) Sketch the line codes corresponding to this sequence for : (i) Non-return to Zero (NRZ), (ii) Alternate Mark Inversion (AMI), (iii) Bi-phase Manchester (BPM), (iv) Differential NRZ (D-NRZ), (v) D-BPM, (vi) Coded Mark Inversion (CMI).

(b) Specify the line coded bit sequence if the following mapping schemes are used (assume : (i) B3ZS, (ii) B4ZS, (iii) B8ZS, and (iv) Pair Selected Ternary (PST) coding for 2 bits patterns.

11. Reading: "Digital Telephony 3rd Ed." by J.C.Bellamy -- pp. 161-188 (Sec. **4.1** thro Sec. **4.3**)