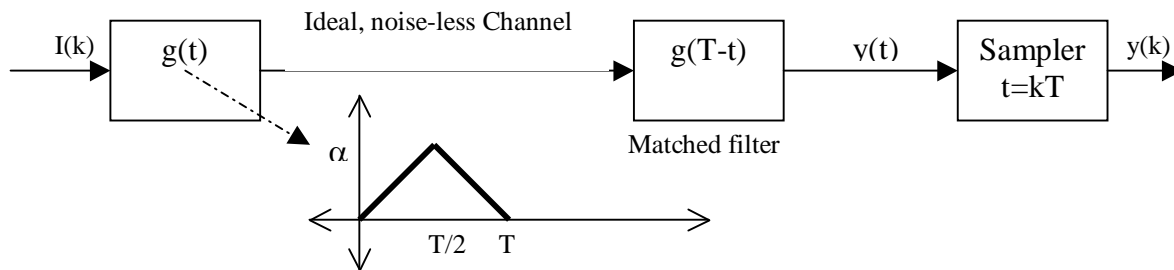


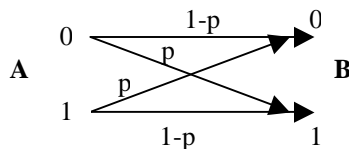
## EC 305 : Communication Systems

### Tutorial 2 : Framing, Matched Filtering, Error Detection, Multi-stage Switching, Erlang-B Formula

1. A single-bit, alternating-bit based framing code is used on a frame with  $N=20$  bits. On the data region, the  $P(0)=0.4$  and  $P(1)=0.6$ , and parallel search is employed to detect the frame boundary. If one declares framing acquisition by processing over  $n=3$  consecutive frames, what is the probability of correct detection of the frame boundary? How many frames  $n$  should one wait if we require this probability to become 0.9999?
2. Consider a  $L$ -bit framing code on a frame of  $N=32$  bits, where the bits are equi-probable. If a multi-bit frame acquisition algorithm is being used:
  - (a) For  $L=4$ , find the expected number of frames to be examined before a particular (randomly picked) frame position mismatches the frame code
  - (b) For the above case, find the average number of bits that pass (i.e., bit durations that elapse) before the frame position is detected.
  - (c) If we need to acquire the frame position in  $1/4^{\text{th}}$  of this time (found in (b)), what should  $L$  be changed to?
3. Problems from "Digital Telephony 3<sup>rd</sup> Ed." by J.C.Bellamy, Chapter 4 (pp.222):**4.2, 4.7\*, 4.8, & 4.9\***.
4. In the base-band digital communication model below, the shape of  $g(t)$  is as shown below. Make a rough but clearly labeled plot of the output of the matched filter,  $y(t)$ , over  $T$  to  $2T$  seconds, if  $I(1) = -1$ ,  $I(2)=1$ , and  $I(3) = -1$  (i.e., plot the output for the *middle bit interval* corresponding to  $I(2)$ ).



5. In a particular discrete measurement model given by  $y(k) = E_g I(k) + n(k)$ , data  $I(k)$  is bi-polar (discrete rv) with equal probability, and noise  $n(k)$  is zero mean Gaussian with variance  $\sigma_n^2 = 0.1$ .
  - (a) If  $E_g = 2$ , get the expression for the probability of bit error  $P_b$  in terms of the  $Q(\cdot)$  or  $\text{erfc}(\cdot)$  functions. Refer mathematical tables for computing these tail probabilities.
  - (b) With this  $P_b = p$ , a binary symmetric channel model (BSC) is defined as below (to model a communication link between A-B).



If the next link B-C also has the same BSC model, then the link from A to C can be constructed by concatenating two of the above BSC models where at the "relay" node B the bits from A are decoded (to 1 or 0), and then reconverted to the appropriate waveform and forwarded to C. Find the probability that a bit sent from A is correctly decoded at C.

6. Given a maximum error burst length of  $L=14$  bits, and a minimum inter-burst time of  $B=250$  bits, define the most efficient matrix parity check code (specify row and column dimensions) which can unambiguously detect all such burst errors. What is the coding efficiency?
7. In the above problem, if  $L$  remains 14, but  $B$  increases to 2500 bits, what is the most suitable design when:
  - (a) Overall memory complexity should be less than 3000

- (b) There is no constraint on memory complexity  
(c) Compare the coding efficiencies obtained in the 2 cases
8. Given a population of  $N=20,000$  users, each offering  $E_u=0.04$  Erlangs of traffic, define a 3-stage blocking switch with  $k$  sub-arrays in the middle-stage, each containing  $250 \times 250$  cross-points such that the blocking probability  $P_b = 10^{-3}$  or less. Use the Lee graph approach to find this least value of  $k$ .  
(a) Determine the number of cross-points for the above switch.  
(b) For the same size of the middle-stage sub-arrays (i.e., same size of  $n$ ) as in (a), define a non-blocking switch. How does the complexity of this switch compare to (a)?  
(c) Rework value of  $k$  and part (a) if we require  $P_b \leq 10^{-6}$ .
9. A total of  $N=4096$  lines have to be switched, where each line offers  $E_u=0.05$  Erlangs of traffic. All the 3 stages of the switch are to be built using sub-arrays of size  $64 \times 64$  (where in 1<sup>st</sup> and 3<sup>rd</sup> stages, not all lines need be utilized if  $k < 64$ ).  
(a) Define a blocking switch such that blocking probability  $P_b = 10^{-3}$  or less. What is its complexity (including unutilized cross-points) ?  
(b) Is it possible to build a non-blocking 3-stage switch in this case? Specify.
10. The first 400 inlets carry users with  $E_u=0.05$  Erlangs while the next 600 inlets carry users with  $E_u=0.01$  Erlangs. Given that the users are grouped into blocks of  $n=50$  each, define a 3-stage block switch with overall  $P_b = 10^{-2}$  or less. What is the total number of cross-points in this switch? *Hint*: The overall  $P_b$  is computed by considering the 4 cases, namely user from set1 calls another user in set1, or user from set1 calls user from set2, etc.
11. Problems from “Digital Telephony 3<sup>rd</sup> Ed.” by J.C.Bellamy, Chapter 5 (pp.274):**5.2, 5.3** (Lee Graph only),**5.4\*thro 5.8\***.
12. Given a switching node where the average number of call arrivals  $\lambda = 10$  per minute:  
(a) What is the probability that 10 or more arrivals occur in a 45 second interval?  
(b) What is the probability that less than 5 arrivals occur in the 45 second interval?
13. What is the amount of traffic  $E$  that can be accepted by  $M=2$  servers if a high blocking probability  $P_b = 0.50$  is allowed? (a) Repeat when the allowed  $P_b = 0.02$ .  
(b) Defining the output utilization factor  $\rho = (1-P_b)E / M$ , what is it for the above 2 cases of  $P_b$ ?
14. Repeat the steps in Pbm. 9 for the case of  $M=3$  servers.
15. Problems from “Digital Telephony 3<sup>rd</sup> Ed.” by J.C.Bellamy, Chapter 12 (pp.568-569):**12.1 thro 12.8, 12.10\*, & 12.13\***.
16. 5<sup>th</sup> Chapter Reading from Bellamy – Sec. **5.1** thro Sec. **5.4.2** (including TSSST switch in Pg.260).
17. 12<sup>th</sup> Chapter Reading from Bellamy -- Sec. **12.1** upto (and not including) Sec. **12.2.2**
18. Reading from URL [ee.iitm.ac.in/~giri/teaching.html](http://ee.iitm.ac.in/~giri/teaching.html): RF Fundamentals. Also see in “Digital Telephony 3<sup>rd</sup> Ed.” by J.C. Bellamy, Sec. 6.4 on Radio System Design (pp. 322 to 326).
19. Reading from “Wireless Communications – Principles & Practice” 2<sup>nd</sup> Ed., by T.S. Rappaport, Chapter 3 – “The Cellular Concept – System Design Fundamentals” pp. 57 to 104. It is also recommended that you (at least) browse thro the first two chapters in this book.