EC-305 Communication Systems

Sept. 2009

Tutorial #1

1. For the QCM signal with magnitude response as below, find the least possible band-pass sampling rate. Make a rough plot of the frequency response of the sampled sequence around 0Hz.



2. In the above problem, assume that the received signal has a phase offset of θ radians; in other words, $s(t) = m_1(t)Cos(2\pi f_c t + \theta) + m_2(t)Sin(2\pi f_c t + \theta)$. Now, what will be the time-domain representation of the sampled sequence? For the special case when $\theta = \pi/2$, what will be the samples of the received signal?

3. A dozen DSB-SC signals of one-sided (low-pass) bandwidth W = 4MHz are present between 800MHz and 896MHz, as shown below. Describe the operations (sampling, rate-conversion, filtering) that you need to do to recover Nyquist rate samples of the 7th DSB-SC signal (i.e., the signal present between 848Mz and 856MHz).



4. From the EC-305 Tutorial #1 of year 2008, solve problems 1, 2, 4, & 5 on uniform quantization.

5. Llyod-Max Non-uniform Quantization: Assume that a random signal x(t) with $x=x(t=t_0)$, follows a probability density function $f_X(x)$. and the objective is to find the *N*-1 signal levels $\{a_1, a_2, \dots, a_{N-1}\}$ and the corresponding *N* quantized values $\{\hat{x}_1, \hat{x}_2, \dots, \hat{x}_N\}$ such that the quantization noise variance given by

$$E_{q} = E[(x - x_{q})^{2}] = \int_{-\infty}^{a_{1}} (x - \hat{x}_{1})^{2} f_{X}(x) dx + \sum_{i=2}^{N-1} \int_{a_{i-1}}^{a_{i}} (x - \hat{x}_{i})^{2} f_{X}(x) dx + \int_{a_{N-1}}^{\infty} (x - \hat{x}_{N})^{2} f_{X}(x) dx$$

is minimized. In other words, differentiate E_q w.r.t $\{a_1, \dots, a_{N-1}\}$ and $\{\hat{x}_1, \dots, \hat{x}_N\}$, equate them to zero, and show the following:

(i)
$$a_i = \frac{\hat{x}_i + \hat{x}_{i+1}}{2}$$
 and (ii) $\hat{x}_i = \frac{\int_{a_{i-1}}^{a_i} x f_x(x) dx}{\int_{a_{i-1}}^{a_i} f_x(x) dx}$

6. The Llyod-Max quantizer indicates that the signal level should be the mid-point of the quantization interval (see (*i*)), and that the quantized value is the "centroid" of the corresponding interval (see (*ii*)). However, a practical method to construct this non-uniform quantizer is given by the iterative method below for a *k*-bit quantizer where $N=2^k$:

<u>Step 1</u>: choose *N*-1 uniform intervals { a_1, a_2, \dots, a_{N-1} }

<u>Step 2</u>: find the corresponding centroids { $\hat{x}_1, \hat{x}_2, \dots, \hat{x}_N$ } using (*ii*)

<u>Step 3</u>: re-compute $\{a_i\}$ using (*ii*) in (*i*)

Iterate between steps 2 and 3 until "convergence"

Use the above procedure for 3 iterations to define the 2-bit LM quantizer for the below source pdf. Compare the E_q obtained between the uniform quantizer (mid-tread) and the LM quantizer.



7. From the EC-305 Tutorial #1 of year 2008, solve problem 6 on non-uniform quantization.

8. From "Digital Telephony" J.C.Bellamy, 3rd Ed., pp. 158-160: Problems **3.1**, **3.2**, **3.3**, **3.4**, **3.5**, **3.8***, **3.14***, **3.16***, **3.17***, **3.18**, and **3.19**.

9. Draw the output of the matched filter over 3 symbol intervals for the bit-sequence 1,-1,1 for each of the below choices of the pulse-shape g(t):



10. For a bit-stream with 8 consecutive bits given by 1,-1,1,-1,-1,-1,1,1, make a rough plot of the following line-codes: (a) NRZ (b) AMI (c) Split-phase Manchester (d) CMI (e) Differential Modulation (f) Differential Split-phase Manchester (g) B3ZS (or HDB3)

11. A state-dependent Miller code of rate 1/2 is defined by the following rule:

(a) input "0" \rightarrow output di-bit is: x 0 where x=0 if preceding input bit is "1" (and "x=1" otherwise) (b) input "1" \rightarrow output di-bit is: 0 1

Use this code on the bit-sequence in Pbm.10. What properties of the Miller code do you observe?