Comm. Systems
(EC-305) Quiz 2 Solutions

20 marks

1. [3 marks]

Avg. frame acquisition time (in bit intervals) $= \frac{N}{2} \cdot \left(AN + 1 \right) \longrightarrow ①$

avg. starting position (of 'N' positions)

$A = \frac{p}{1-p}$, $p = \frac{1}{2^{L_1}}$

$= \frac{1}{2^{L_1}-1} = \frac{1}{2^6-1} = \frac{1}{63}$; -1-

For $N = 100$,

Avg. time $= 50 \cdot \left(\frac{100}{63} + 1 \right)$ -1-

$S = 129.3 \approx \boxed{129}$ bit intervals

Now, for no choice of L_2 -1- can we get a ten fold reduction in S to $S/10$ (i.e. $12.9 \approx 13$ bits) since a "floor" of $N/2 = 50$ bits always in ①.

Note: Only for large N is this possible

2. [3+4=7 marks]

(a) Parallel search is easiest when 2 frames worth of bits are taken, i.e., 150 bits.

Acquisition time (for 10 consecutive hits of "1010…") $= 150 \times 5 = \boxed{750 \text{ bits}}$ -3-

(b) Serial search

(b1) Best Case: Start exactly at beginning of short frame (50 bit) at the first bit (ie, bit "0")

$\Rightarrow \underset{0}{\uparrow} 50 + \underset{1}{\uparrow} 100 + \underset{0}{\uparrow} 50 + \underset{1}{\uparrow} 100 + \underset{0}{\uparrow} 50 + \underset{1}{\uparrow} 100 + \underset{0}{\uparrow} 50 + \underset{1}{\uparrow} 100 + \underset{0}{\uparrow} 50 + \underset{1}{\uparrow} \Rightarrow \boxed{650 \text{ bits}}$ $-1\frac{1}{2}-$

Note: Starting on the long frame would have given 700 bits

(b2) Worst Case:

$\text{OR}\left\{\begin{array}{l} 0 \xrightarrow{50} 1 \xrightarrow{100} 0 \to 1 \to 0 \to 1 \to 0 \to 1 \xrightarrow{50} \overset{\times}{0}\ (1) \\ 1 \xrightarrow{100} 0 \xrightarrow{50} 1 \to 0 \to \cancel{1} \to 0 \to 1 \to 0 \xrightarrow{100} \overset{\times}{1}(0) \end{array}\right\} 600$

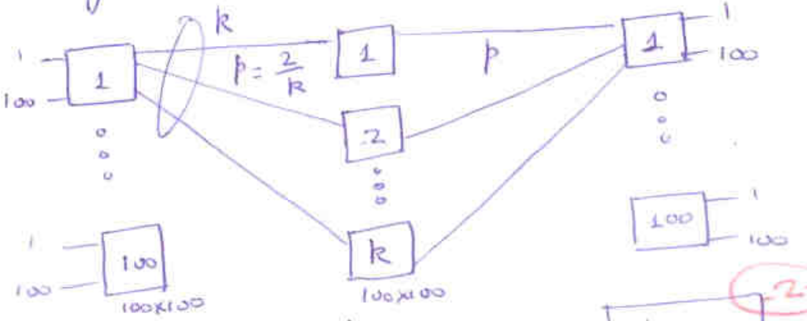
After just missing bit "1" in long frame (100 bits), we will have in the worst case 99 × 600 bits to elapse till we arrive at bit "0" in the beginning of short frame. Will take 650 bits (as in b1) after that (for revised question)

$\therefore$ total $= 99 \times 600 + 650 = \boxed{60{,}050 \text{ bits}}$ $-2\frac{1}{2}-$

EC-305 Quiz 2 → solns. contd.

3. [4 marks]

3-stage blocking switch for $N = 10{,}000$ users

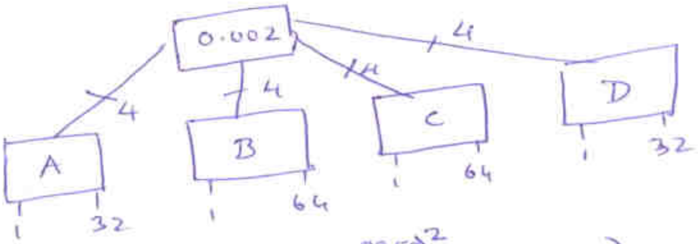


$p = \frac{100 \times 0.02 \leftarrow Eu}{k} = \frac{2}{k}$;

$P_b = \left(1 - (1-p)^2\right)^k$ and for $\boxed{k = 12}$ (-2-) $\Rightarrow 6.62 \times 10^{-7} < 10^{-6}$

Total # of 100×100 ~~switch~~ sub-arrays $= 2 \times 100 + k = \boxed{212}$ (-2-);

4. [6 marks]



(a) A & D → $\boxed{P_b = 0.0952}$ (-1-)

B & C → $\boxed{P_b = 0.3106}$ (-1-)

i.e, $= \boxed{0.817}$ (-1-)

(b) $P[A \rightarrow D] = (1 - \overset{0.0952}{\cancel{0.3106}})^2 (1 - 0.002) =$

(c) $P[A \overset{\text{cannot call}}{\not\rightarrow} C] = 1 - \left[(1 - 0.3106)(1 - 0.002)(1 - 0.0957)\right] = \boxed{0.3775}$. (-1-)

(d) To find overall average blocking Probability P_b, let us define $P[D \not\rightarrow C] = \boxed{\overset{0.3775}{\cancel{0.183}};}$

$p_1 \triangleq P[A \not\rightarrow B] = P[A \not\rightarrow C] = P[D \not\rightarrow B] = P[D \not\rightarrow C]$

$p_2 \triangleq P[A \not\rightarrow D] = \boxed{0.183;}$

$p_3 \triangleq P[B \not\rightarrow C] = (1 - 0.3106)^2 (1 - 0.002) = \boxed{0.5258;}$ ∴

$P_b = \underbrace{P\left[\text{Blocking} \middle| \begin{matrix}\text{Call originated} \\ \text{in } A\end{matrix}\right]} P\left[\begin{matrix}\text{call orginated} \\ \text{in } A\end{matrix}\right]_{\hookrightarrow 1/6} + \cdots\cdots \left\{ \sum_{i=1}^{4} P[Bl | A_i] P(A_i) \right.$ $\{\frac{1}{6}, \frac{2}{6}, \frac{2}{6}, \frac{1}{6}\}$

$P\left[Bl. \middle| \begin{matrix}\text{call from} \\ A\end{matrix}\right] = \underbrace{P\left[Bl \middle| \begin{matrix}\text{call from} \\ A\end{matrix}, A \text{ calls } B\right]}_{p_1} \underbrace{P[A \text{ calls } B]}_{2/5} + \underset{\swarrow p_1}{P\left[Bl \middle| \begin{matrix}\text{call from} \\ A\end{matrix}, A \text{ calls } C\right]} . \, P[A \text{ calls } C]_{\hookrightarrow 2/5}$

& so on $+ \underbrace{P\left[Bl. \middle| \begin{matrix}\text{call from} \\ A\end{matrix}, A \text{ calls } D\right]}_{p_2} P[A \text{ calls } D]_{\hookrightarrow 1/5}$

Finally, using symmetry we get

$P_b = \overset{A \& D}{\hookrightarrow} 2 \times \frac{1}{6}\left[\frac{2}{5} \times p_1 \times 2 + \frac{1}{5} \times p_2\right] + \overset{B \& C}{\overset{\downarrow}{2}} \times \frac{2}{6}\left[\frac{1}{4} \times p_1 \times 2 + \frac{2}{4} p_3\right]$

$= \underset{(3/5)}{\frac{9}{15}} p_1 + \frac{1}{15} p_2 + \underset{(1/3)}{\frac{5}{15}} p_3 = \boxed{0.4139}$ (-2-)